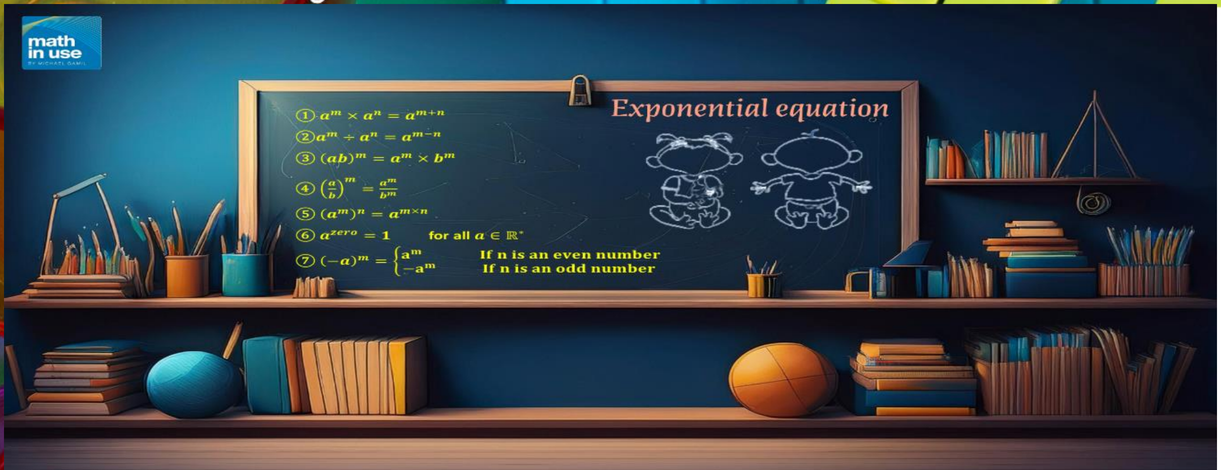

Rational exponents and exponential equations

Choose the correct answer

$a^m \times a^m = \dots a^{m+m} \dots = a^{2m}$

(a) a^{m^2} (b) a^{2m} (c) $2a^m$ (d) ma^2

Choose the correct answer

$\sqrt[5]{a^3} \times \sqrt{a^3} = \dots\dots\dots\dots$

(a) $\sqrt[7]{a^3}$ (b) $\sqrt[7]{a^6}$ (c) $\sqrt[7]{a^{14}}$ (d) $a^2 \sqrt[10]{a}$

$a^{\frac{3}{5}} \times a^{\frac{3}{2}} = a^{\frac{21}{10}} = \sqrt[10]{a^{21}} = a^2 \sqrt[10]{a}$

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Choose the correct answer

If $2^{x+1} = 8$, then $x =$ ……………

(a) 1 (b) 2 (c) 3 (d) 4

$2^{x+1} = 2^3$

$x + 1 = 3$

$\boxed{x = 2}$

Choose the correct answer

If $3^{x+5} = \frac{1}{27}$, then $x =$ ……………

(a) – 3 (b) 8 (c) – 8 (d) 3

$3^{x+5} = 3^{-3}$

$x + 5 = -3$

$\boxed{x = -8}$

Choose the correct answer

If $5^{x-1} = 4^{x-1}$, then $x = \dots\dots\dots\dots$

(a) 5 (b) 1 (c) -1 (d) zero

$x - 1 = 0$

$\boxed{x = 1}$

Choose the correct answer

The solution set of the equation : $5^{x^2-4} = 7^{x^2-4}$ is ....

(a) $\{2\}$ (b) $\{-2\}$ (c) $\{2, -2\}$ (d) $\{\text{zero}\}$

$x^2 - 4 = 0$

$x^2 = 4$

$\boxed{x = \pm 2}$

S.S. = $\{2, -2\}$

Choose the correct answer

If $7^{x+1} = 3^{2x+2}$, then $5^{x+1} = \ldots\ldots\ldots\ldots$

(a) zero (b) 1 (c) 2 (d) 5

$7^{x+1} = 3^{2(x+1)}$

$7^{x+1} = 9^{x+1}$

$\therefore \boxed{x+1=0}$

$5^{x+1} = 5^{0} = 1$

Choose the correct answer

If $\left(\frac{1}{2}\right)^{a^2 - a - 2} = 1$ where a > zero , then a = ...

(a) 1 (b) –3 (c) 2 (d) 3

$a^2 - a - 2 = 0$

$a = 2$ $a = -1$ (ref.)

Choose the correct answer

The solution set of the equation : $7^{x^2} = 49^{x+4}$ is ....

(a) $\{-2\}$ (b) $\{-2, 4\}$ (c) $\{-2, 3\}$ (d) $\{2, -4\}$

$7^{x^2} = (7^2)^{x+4}$

$7^{x^2} = 7^{2x+8}$

$x^2 = 2x + 8$

$x^2 - 2x - 8 = 0$ S.S. $= \{4, -2\}$

$x = 4$ $x = -2$

Choose the correct answer

If $3^{x} = 2$, $2^{y} = 9$, then $x\,y =$ ....

(a) 2 (b) 3 (c) 8 (d) 18

$\boxed{2}^{y} = 9$

$(3^{x})^{y} = 3^{2}$

$3^{xy} = 3^{2} \Longrightarrow xy = 2$

Choose the correct answer

If $5^x = 2$, then $(25)^x =$ ....

(a) 10 (b) 625 (c) 4 (d) 2

$(25)^x = (5^2)^x = (5^x)^2 = 2^2 = 4$

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Choose the correct answer

If $2^x = 5$, then $2^{x+2} = \ldots\ldots\ldots\ldots\ldots$

(a) 15 (b) 4 (c) 10 (d) 20

$$2^{x+2} = 2^x \times 2^2$$

$$= 5 \times 4 = 20$$

Choose the correct answer

If $x^{\frac{3}{2}} = 64$, then $x =$ ……………

(a) 512　　(b) 16　　(c) 4　　(d) 2

$$\left(x^{\cancel{\frac{3}{2}}}\right)^{\cancel{\frac{2}{3}}} = (64)^{\frac{2}{3}}$$

$$x = 16$$

Choose the correct answer

If $x^{\frac{2}{5}} = 4$, then $x =$ ....

(a) 4 (b) 16 (c) ± 4 (d) ± 32

$$\left(x^{\frac{2}{5}}\right)^{\frac{5}{2}} = \pm(4)^{\frac{5}{2}}$$

$$x = \pm 32$$

Choose the correct answer

If $4x^5 = 128$, then $x =$ ....

(a) 4 (b) ± 2 (c) 2 (d) -2

$x^5 = 128 \div 4$

$x^5 = 32$

$x = \sqrt[5]{32}$

$\boxed{x = 2}$

$x^5 = 2^5$

$\boxed{x = 2}$

$(x^5)^{\frac{1}{5}} = (32)^{\frac{1}{5}}$

$\boxed{x = 2}$

Choose the correct answer

$(128)^{-\frac{2}{7}} = \dots\dots\dots\dots$

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 4

$(2^{\cancel{7}})^{-\frac{2}{\cancel{7}}} = 2^{-2} = \left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$

128	2
64	2
32	2
16	2
8	2
4	2
2	2
1	

Choose the correct answer

$\sqrt[4]{(16)^{-3}} = \ldots\ldots\ldots\ldots\ldots$

(a) 8 (b) -8 (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

$(16)^{-\frac{3}{4}} = (2^{\cancel{4}})^{-\frac{3}{\cancel{4}}} = (2)^{-3} = \left(\frac{1}{2}\right)^{3}$

$= \frac{1}{8}$

Choose the correct answer

If x , y $\in \mathbb{R}$, then $\sqrt{x^2 y^6} = \dots |xy^3|$

(a) $x y^2$ (b) $|x y^3|$ (c) $\frac{1}{2} x^2 y^6$ (d) $\pm x y^3$

$\boxed{y^2}$

$(-2)^2 = (2)^2 = 4$

$(-2)^3 \neq (2)^3$

-8 8

$\sqrt{(2)^2} = \boxed{2}$

$\sqrt{(-2)^2} = \boxed{2}$

Choose the correct answer

$2xy^2$

$\sqrt[4]{16x^4y^8} = \dots 2|x|y^2$

(a) $2xy^2$ (b) $2|x|y^2$ (c) $2x|y|^2$ (d) $2x|y^2|$

Choose the correct answer

If $2^{x-1} = 44$, then $2^{x-2} =$ ....

(a) 18 (b) 22 (c) 10 (d) 16

$2^{x-1} = 44$

$2^{x} \times 2^{-1} = 44$

$2^{x} = 44 \div 2^{-1}$

$\boxed{2^{x} = 88}$

$2^{x-2} = 2^{x} \times 2^{-2}$

$= 88 \times \frac{1}{4}$

$= 22$

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Choose the correct answer

If $x^{\frac{5}{3}} = 2y^{\frac{4}{3}} = 32$, then $x + y =$ ....

(a) 16 (b) zero. (c) 16 , – 16 (d) zero , 16

$x^{\frac{5}{3}} = 32$

$\left(x^{\frac{5}{3}}\right)^{\frac{3}{5}} = (32)^{\frac{3}{5}}$

$\boxed{x = 8}$

$2y^{\frac{4}{3}} = 32$

$\left(y^{\frac{4}{3}}\right)^{\frac{3}{4}} = \pm(16)^{\frac{3}{4}}$

$\boxed{y = \pm 8}$

$x + y =$

$8 + 8 = 16$

$8 + (-8) = \text{zero}$

Choose the correct answer

The solution set of the equation : $3^{x+1} + 3^{x} = 12$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\{3\}$ (c) $\{1\}$ (d) $\{1, 0\}$

$$(3^{x} \times 3^{1}) + (3^{x}) = 12$$

$$3^{x} [3 + 1] = 12$$

$$3^{x} = 12 \div 4$$

$$3^{x} = 3^{1} \Rightarrow \boxed{x = 1}$$

Choose the correct answer

The solution set of the equation : $3^{x} + 3^{3-x} = 12$ is ……………

(a) $\{1, 2\}$ (b) $\{0, 3\}$ (c) $\{3, 4\}$ (d) $\{-1, -2\}$

$$(3^x) + (3^3 \times 3^{-x}) = 12$$

$$3^x + \frac{27}{3^x} = 12$$

let $3^x = y$

$$y + \frac{27}{y} = 12$$

$\boxed{\times y}$

$$y^2 + 27 = 12y$$

$$y^2 - 12y + 27 = 0$$

$y = 3$ $y = 9$

$3^x = 3^1$ $3^x = 3^2$

$\boxed{x = 1}$ $\boxed{x = 2}$

$$S.S. = \{1, 2\}$$

Choose the correct answer

The solution set of the equation : $\sqrt[3]{x^2} - 3\sqrt[3]{x} + 2 = 0$ is ....

(a) $\{1, 8\}$ (b) $\{9, 3\}$ (c) $\{8\}$ (d) $\{1\}$

$x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 2 = 0$

let $x^{\frac{1}{3}} = y$

$x^{\frac{2}{3}} = y^2$

$y^2 - 3y + 2 = 0$

$y = 1$ $y = 2$

$\left(x^{\frac{1}{3}}\right)^3 = (1)^3$ $\left(x^{\frac{1}{3}}\right)^3 = (2)^3$

$\boxed{x = 1}$ $\boxed{x = 8}$

S.S. = $\{1, 8\}$

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Choose the correct answer

The solution set of the equation : $9^x - 30 \times 3^{x-1} + 9 = 0$ is ....

(a) $\{0, 1\}$ (b) $\{1, 2\}$ (c) $\{0, 2\}$ (d) $\{0, 3\}$

$(3^2)^x - 30 \times 3^x \times 3^{-1} + 9 = 0$

$3^{2x} - 10 \times 3^x + 9 = 0$

let $3^x = y$

$3^{2x} = y^2$

$y^2 - 10y + 9 = 0$

$y = 1$

$3^x = 1$

$\boxed{x = 0}$

$y = 9$

$3^x = 9$

$3^x = 3^2$

$\boxed{x = 2}$

$S.S. = \{0, 2\}$

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Choose the correct answer

The number of real roots of the equation : $x^n = a$ where n is an odd number is ....

$\in \mathbb{R}$

(a) 1 (b) 2 (c) 3 (d) n

$x^3 = 8$

$x = \sqrt[3]{8}$

$x^5 = 32$

$x = \sqrt[5]{32}$

* $x^n = a$

n roots

* $x^n = a$ n=odd

1 real sol.

rest im.

* $x^n = a$ n=even

2 real sol.

rest im.

Choose the correct answer

$6 \Rightarrow$ even

The number of real roots of the equation : $x^6 = a$ where $a > 0$, is ……………

(a) 1 (b) 2 (c) 3 (d) 6

Choose the correct answer

The number of roots of the equation : $x^3 = 4$ is ……………

(a) 1 (b) 2 (c) 3 (d) 4

Choose the correct answer

The number of real roots of the equation : $x^4 = -16$ is ....

(a) zero (b) 1 (c) 2 (d) 4

$$x = \pm \sqrt[4]{-16}$$

$$= \pm 2i$$

Choose the correct answer

The set of the real roots of the equation : $(x-2)^4 = 16$ equals ....

(a) $\{0\}$ (b) $\{4\}$ (c) $\{8\}$ (d) $\{0, 4\}$

$(x-2)^4 = 2^4$

$x - 2 = \pm 2$

$x - 2 = 2$ or $x - 2 = -2$

$\boxed{x = 4}$ $\boxed{x = 0}$

S.S. = $\{0, 4\}$

Choose the correct answer

The solution set of the equation : $(x-3)^{\frac{5}{3}} = 32$ in $\mathbb{R}$ is ....

(a) $\{2\}$ (b) $\{11\}$ (c) ~~$\{11, -5\}$~~ (d) ~~$\{-11, 11\}$~~

$$\left((x-3)^{\cancel{\frac{5}{3}}}\right)^{\cancel{\frac{3}{5}}} = (32)^{\frac{3}{5}}$$

$$x-3 = 8$$

$$\boxed{x = 11}$$

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Choose the correct answer

If $x \in \mathbb{R}^*$, n is an even integer , which of the following is true ?

(a) $x^n > 0$ (b) $x^n < 0$ (c) $x^n \le 0$ (d) $x^n = 0$

$(2)^2 = 4$ $(-2)^2 = 4$ $(-2)^0 = 1$

$(2)^{-2} = \frac{1}{4}$ $(-2)^{-2} = \frac{1}{4}$

Choose the correct answer

If $x \in \mathbb{R}^-$, n is an odd integer , which of the following is true ?

(a) $x^n > 0$ (b) $x^n < 0$ (c) $x^n \le 0$ (d) $x^n = 0$

$(-2)^3$

$= -(2)^3$

$= -8$

$\left(-\frac{1}{2}\right)^3$

$= -\left(\frac{1}{2}\right)^3$

$= -\frac{1}{8}$

Choose the correct answer

Which of the following is not equal to $\sqrt[5]{x^4}$?

(a) $\left(\sqrt[5]{x}\right)^4$ (b) $\sqrt[4]{x^5}$ (c) $x^{\frac{4}{5}}$ (d) $\left(x^{\frac{1}{5}}\right)^4$

$$\sqrt[5]{x^4} = x^{\frac{4}{5}} = \left(x^4\right)^{\frac{1}{5}} = \left(x^{\frac{1}{5}}\right)^4$$

$$\downarrow$$

$$\left(\sqrt[5]{x}\right)^4$$

Choose the correct answer

$(-5)-(3)$

If $a < b < 0 < c$, then $\dfrac{\sqrt[4]{b^4 c^4} + b\sqrt{(a-c)^2}}{\sqrt{a^2 b^2}} = \dots\dots\dots\dots\dots$

(a) 1 (b) -1 (c) $\dfrac{a}{b}$ (d) $\dfrac{-c}{2}$

$$\frac{\sqrt[4]{(-1)^4(1)^4} + (-1)\sqrt{(-2-1)^2}}{\sqrt{(-2)^2(-1)^2}}$$

$$\frac{1-3}{2} = \frac{-2}{2}$$

$$= -1$$

Another sol.

$$\frac{|b||c| + b\,|a-c|}{|a||b|}$$

$$\frac{(-b)(c) - b\,(a-c)}{(-a)(-b)}$$

$$= \frac{\cancel{-bc} - ab + \cancel{bc}}{ab}$$

$$= \frac{-ab}{ab} = -1$$

Choose the correct answer

If $\overset{-}{a} < 0 < \overset{+}{b} < \overset{+}{c}$, then which of the following does not belong to $\mathbb{R}$?

(a) $\sqrt[3]{ab}$ (b) $\sqrt[4]{bc}$ (c) $\sqrt[5]{ab+c}$ (d) $\sqrt[6]{ac}$

$\sqrt[3]{-ve}$ $\sqrt[4]{+ve}$ $\sqrt[5]{-ve+ve}$ $\sqrt[6]{-ve}$

R R R $\sqrt[even]{-ve}$

Im.

Choose the correct answer

If $\sqrt{2} \times \sqrt[3]{3} = \sqrt[6]{x}$, then $x =$ ....

(a) 27 (b) 48 (c) 72 (d) 108

$$\sqrt[6]{x} = \sqrt{2} \times \sqrt[3]{3}$$

$$\left[x^{\frac{1}{6}}\right]^6 = \left[2^{\frac{1}{2}}\right]^6 \times \left[3^{\frac{1}{3}}\right]^6$$

$$x = 2^3 \times 3^2$$

$$x = 8 \times 9 = 72$$

Choose the correct answer

$\sqrt{9^{16x^2}} =$ ....

(a) 3^{4x} (b) 3^{8x^2} (c) 9^{4x} (d) 9^{8x^2}

$$\left[9^{16x^2}\right]^{\frac{1}{2}} = 9^{8x^2}$$

Choose the correct answer

If $3^{x-2} = \sqrt[4]{27}$, then $x =$ ....

(a) $\frac{11}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) 6

$$3^{x-2} = \sqrt[4]{3^3}$$

$$3^{x-2} = 3^{\frac{3}{4}}$$

$$x - 2 = \frac{3}{4} \Rightarrow x = \frac{3}{4} + 2$$

$$x = \frac{11}{4}$$

Choose the correct answer

If $3^{x+2} = 6^{x-1}$, then $2^x = \ldots\ldots\ldots\ldots$

(a) 54 (b) 27 (c) $\frac{1}{9}$ (d) $\frac{1}{36}$

$$3^{x+2} = (2 \times 3)^{x-1}$$

$$3^{x+2} = 2^{x-1} \times 3^{x-1}$$

$$\frac{3^{x+2}}{3^{x-1}} = 2^{x-1}$$

$$3^{\cancel{x}+2-\cancel{x}+1} = 2^x \times 2^{-1}$$

$$3^3 = 2^x \times \frac{1}{2}$$

$$27 \div \frac{1}{2} = 2^x$$

$$\boxed{2^x = 54}$$

Choose the correct answer

If $2^{x} = 20$, $n < x < n + 1$, n is an integer , then n = ...4......

(a) 4 (b) 5 (c) 6 (d) 7

2^{4} 2^{x} 2^{5}

16 20 32

$4 < x < 5$

Choose the correct answer

If $3^{x} < 0$, then ....

(a) $0 < x < 1$ ✗

(b) $-1 < x < 0$ ✗

(c) $x < -1$ ✗

(d) there are no values for x satisfy this inequality.

ⓐ $3^{\frac{1}{2}} = \sqrt{3} \Rightarrow +ve$

ⓑ $3^{-\frac{1}{2}} = \left(\frac{1}{3}\right)^{\frac{1}{2}} = \sqrt{\frac{1}{3}} = +ve$

ⓒ $3^{-2} = \left(\frac{1}{3}\right)^{2} = \frac{1}{9} = +ve$

Choose the correct answer

The number $\left(2^{24} + 2^{23} + 2^{22}\right)$ is divisible by ....

(a) 3 (b) 5 (c) 7 (d) 9

$4 + 2 + 1$

$2^{22}\left[2^{2} + 2^{1} + 1\right]$

$2^{22} \times 7$

Choose the correct answer

Study

If $3^a = 4^b$, then $9^{\frac{a}{b}} + 16^{\frac{b}{a}} =$ ....

(a) 7 (b) 12 (c) 20 (d) 25

$$9^{\frac{a}{b}} + 16^{\frac{b}{a}} = (3^2)^{\frac{a}{b}} + (4^2)^{\frac{b}{a}}$$

$$= (3^a)^{\frac{2}{b}} + (4^b)^{\frac{2}{a}} = (4^b)^{\frac{2}{b}} + (3^a)^{\frac{2}{a}}$$

$$= 4^2 + 3^2 = 16 + 9 = 25$$

Choose the correct answer

If $2^a = 3$, $3^b = 7$, $7^c = 11$, then $2^{abc} = \dots\dots\dots\dots$

(a) 11 (b) 27 (c) 21 (d) 231

$$2^{abc} = (2^a)^{bc} = \left[(2^a)^b\right]^c = (3^b)^c$$

$$= 7^c = 11$$

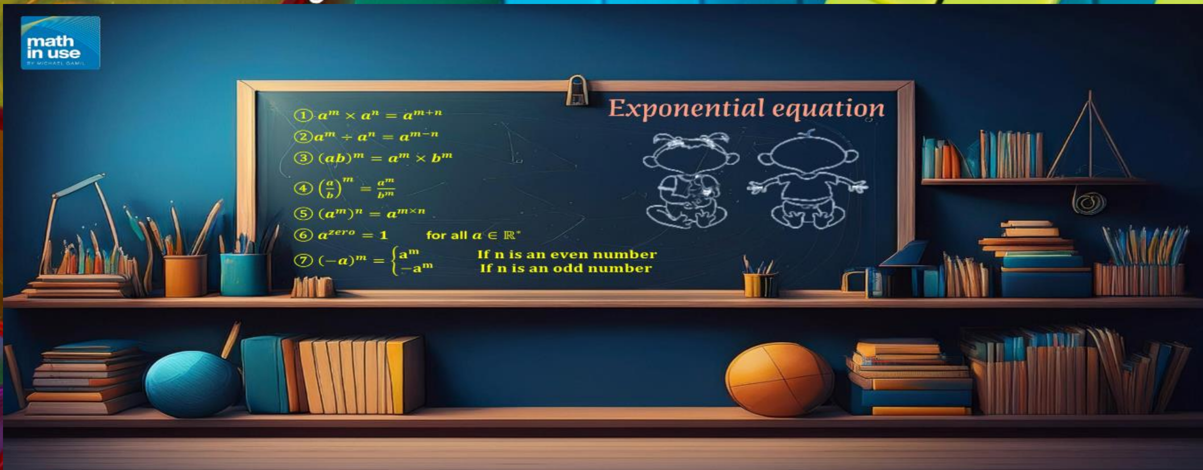
Choose the correct answer

If $2^x = a$, $3^x = b$, $5^x = c$, then $(90)^x =$ ……………

(a) abc (b) a^2bc (c) ab^2c (d) $a + 2b + c$

$$(90)^x = (2^1 \times 3^2 \times 5^1)^x$$
$$= 2^x \times 3^{2x} \times 5^x$$
$$= 2^x \times (3^x)^2 \times 5^x$$
$$= ab^2c$$

90	2
45	3
15	3
5	5
1	



Rational exponents and exponential equations

Answer each of the following questions

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

$\boxed{1}$ $x^{\frac{7}{2}} = 128$

$$\left(x^{\frac{7}{2}}\right)^{\frac{2}{7}} = (128)^{\frac{2}{7}}$$

$x = 4$ S. S. in R = {4}

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

2 $\sqrt[3]{(x-1)^5} = 32$

$(x-1)^{\frac{5}{3}} = 32$

$\left[(x-1)^{\frac{5}{3}}\right]^{\frac{3}{5}} = [32]^{\frac{3}{5}}$

$x-1 = 8$

$x = 8+1 = 9$

S.S. in $\mathbb{R} = \{9\}$

1. **Find in $\mathbb{R}$ the solution set of each of the following equations :**

3. $(x+1)^{-\frac{5}{2}} = (32)^{-\frac{1}{2}}$

$$\left[(x+1)^{-\frac{5}{2}}\right]^{-\frac{2}{5}} = \left[(32)^{-\frac{1}{2}}\right]^{-\frac{2}{5}}$$

$$x+1 = (32)^{\frac{1}{5}}$$

$$x+1 = 2$$

$$x = 2-1$$

$$\boxed{x=1}$$

S. S. in $\mathbb{R} = \{1\}$

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

4 $(\sqrt{x}+2)^{\frac{1}{2}}=3$

$$\left[(\sqrt{x}+2)^{\frac{1}{2}}\right]^2=[3]^2$$

$$\sqrt{x}+2=9$$

$$\sqrt{x}=7$$

$$\left[x^{\frac{1}{2}}\right]^2=[7]^2 \Rightarrow x=49$$

S.S. in R = {49}

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

5 $x^{\frac{4}{3}} - 10\,x^{\frac{2}{3}} + 9 = 0$

let $x^{\frac{2}{3}} = y$

$x^{\frac{4}{3}} = y^2$

$y^2 - 10y + 9 = 0$

$y = 1$

$x^{\frac{2}{3}} = 1$

$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm(1)^{\frac{3}{2}}$

$x = \pm 1$

$y = 9$

$x^{\frac{2}{3}} = 9$

$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm(9)^{\frac{3}{2}}$

$x = \pm 27$

S. S. in R $= \{\pm 1, \pm 27\}$

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

6 $x + 15 = 8\sqrt{x}$

let $x^{\frac{1}{2}} = y$

$x^{1} = y^{2}$

$x^{1} + 15 = 8x^{\frac{1}{2}}$

$y^{2} + 15 = 8y$

$y^{2} - 8y + 15 = 0$

$y = 3$ $\qquad$ $y = 5$

$\left(x^{\frac{1}{2}}\right)^{2} = (3)^{2}$ $\qquad$ $\left(x^{\frac{1}{2}}\right)^{2} = (5)^{2}$

$\boxed{x = 9}$ $\qquad$ $\boxed{x = 25}$

S.S. in R = {9, 25}

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

7 $\sqrt[5]{x^4} - 3\sqrt[5]{x^2} = 4$

let $x^{\frac{2}{5}} = y$

$x^{\frac{4}{5}} = y^2$

$x^{\frac{4}{5}} - 3x^{\frac{2}{5}} - 4 = 0$

$y^2 - 3y - 4 = 0$

$y = 4$ $\qquad$ $y = -1$

$x^{\frac{2}{5}} = 4$ $\qquad$ $x^{\frac{2}{5}} = -1$ ref.

$\left(x^{\frac{2}{5}}\right)^{\frac{5}{2}} = \pm(4)^{\frac{5}{2}}$

$x = \pm 32$

$S.S. = \{\pm 32\}$

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① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

8 $5^{2x-1} = \frac{1}{125}$

125	5
25	5
5	5
1	

$5^{2x-1} = 5^{-3}$

$2x-1 = -3$

$2x = -3+1$

$2x = -2$

$\boxed{x = -1}$

S.S. in R = $\{-1\}$

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

9 $2^{x^2-9} = 1$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm\sqrt{9} = \pm 3$

S.S. = $\{\pm 3\}$

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

10 $3^{|3x-4|} = 9^{2x-2}$

$3^{|3x-4|} = (3^2)^{2x-2}$

$3^{|3x-4|} = 3^{4x-4}$

$|3x-4| = 4x-4$

$x \geqslant \frac{4}{3}$

$3x-4 = 4x-4$

$3x-4x = -4+4$

$-x = 0$

$\boxed{x=0}$ ref.

$x < \frac{4}{3}$ 1.33

$-3x+4 = 4x-4$

$-3x-4x = -4-4$

$-7x = -8$

$x = \frac{-8}{-7}$

$x = \frac{8}{7} = 1.14$

S.S. in R $= \{\frac{8}{7}\}$

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

11 $5^{x-1} \times 7^{1-x} = \frac{25}{49}$

$$5^{x} \times 5^{-1} \times 7^{1} \times 7^{-x} = \frac{25}{49}$$

$$5^{x} \times \frac{1}{5} \times \frac{7}{7^{x}} = \frac{25}{49}$$

$$\frac{5^{x}}{7^{x}} \times \frac{7}{5} = \frac{25}{49}$$

$$\left(\frac{5}{7}\right)^{x} = \frac{25}{49} \div \frac{7}{5}$$

$$\left(\frac{5}{7}\right)^{x} = \frac{125}{343}$$

$$\left(\frac{5}{7}\right)^{x} = \left(\frac{5}{7}\right)^{3}$$

$$\boxed{x = 3}$$

S'. S'. in R = {3}

math in use
BY MICHAEL GAMIL

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

12 $\dfrac{12^{3x-2} \times 9^{x+1}}{18^{2x} \times 4^{2x-2}} = 9$

S

D

C

$$\begin{array}{r|l} 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

$$\frac{(2^2 \times 3^1)^{3x-2} \times (3^2)^{x+1}}{(2^1 \times 3^2)^{2x} \times (2^2)^{2x-2}} = 9$$

$$\frac{2^{6x-4} \times 3^{3x-2} \times 3^{2x+2}}{2^{2x} \times 3^{4x} \times 2^{4x-4}} = 9$$

$$2^{\cancel{6x}-4-\cancel{2x}-\cancel{4x}+4} \times 3^{3x-\cancel{2}+2x+\cancel{2}-4x} = 3^2$$

$$\cancel{2^0} \times 3^x = 3^2$$

$$3^x = 3^2$$

$$\boxed{\therefore x = 2}$$

S.S. in $\mathbb{R}$ = $\{2\}$

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

13 $\left(\sqrt{3}\right)^{x^2-5x} = 1$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0$ $x = 5$

S.S. in R = $\{0, 5\}$

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

14 $5^{x^2} = 25^{x+4}$

$$5^{x^2} = (5^2)^{x+4}$$

$$5^{x^2} = 5^{2x+8}$$

$$x^2 = 2x + 8$$

$$x^2 - 2x - 8 = 0$$

$x = 4$ $x = -2$

S.S. in R = {−2, 4}

① **Find in ℝ the solution set of each of the following equations :**

15 $\left(\sqrt{7}\right)^{|x+2|} = 49$

$\left(7^{\frac{1}{2}}\right)^{|x+2|} = 7^2$

$7^{\frac{1}{2}|x+2|} = 7^2$

$\frac{1}{2}|x+2| = 2$ $\boxed{\times 2}$

$|x+2| = 4$

$x \geqslant -2$ $x < -2$

$x + 2 = 4$ $x + 2 = -4$

$x = 2$ $x = -6$

$S.S = \{-6, 2\}$

① **Find in $\mathbb{R}$ the solution set of each of the following equations :**

16 $\sqrt{9^x - 2 \times 3^{x+1} + 9} = 24$

let $3^x = y$

$3^{2x} = y^2$

$$\sqrt{3^{2x} - 2 \times 3^x \times 3 + 9} = 24$$

$$\sqrt{y^2 - 6y + 9} = 24$$

$$\sqrt{(y-3)^2} = 24$$

$$|y-3| = 24$$

$y \geq 3$

$y - 3 = 24$

$y = 27$

$3^x = 27$

$3^x = 3^3 \Rightarrow \boxed{x = 3}$

$y < 3$

$y - 3 = -24$

$y = -21$

$3^x = -21$ (ref.)

S.S. in R $= \{3\}$

② **Find in $\mathbb{R}$ the solution set of each of the following equations :**

1 $5^{x+1} + 5^{x-1} = 26$

$$(5^x \times 5^1) + (5^x \times 5^{-1}) = 26$$

$$5^x \times [5 + \frac{1}{5}] = 26$$

$$5^x \times \frac{26}{5} = 26$$

$$5^x = 26 \div \frac{26}{5}$$

$$5^x = 5^1 \Rightarrow \boxed{x = 1}$$

S.S. in R = {1}

② **Find in $\mathbb{R}$ the solution set of each of the following equations :**

2 $7^{2-x} + 7^{-x} = 50$

$(7^2 \times 7^{-x}) + (7^{-x}) = 50$

$7^{-x}[7^2 + 1] = 50$

$7^{-x} \times 50 = 50$

$7^{-x} = 1$

$-x = 0$

$\boxed{x = 0}$

S.S. in R = {0}

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② **Find in $\mathbb{R}$ the solution set of each of the following equations :**

3 $2^{x} + 2^{5-x} = 12$

$$(2^{x}) + (2^{5} \times 2^{-x}) = 12$$

$$2^{x} + \frac{2^{5}}{2^{x}} = 12$$

let $\boxed{2^{x} = y}$

$$y + \frac{32}{y} = 12$$

$\times y$

$$y^{2} + 32 = 12y$$

$$y^{2} - 12y + 32 = 0$$

$y = 8$ $y = 4$

$2^{x} = 2^{3}$ $2^{x} = 2^{2}$

$\boxed{x = 3}$ $\boxed{x = 2}$

$S.S. = \{3, 2\}$

② **Find in $\mathbb{R}$ the solution set of each of the following equations :**

4 $\left(\frac{1}{2}\right)^{x+1}+\left(\frac{1}{2}\right)^{x+3}+\left(\frac{1}{2}\right)^{x+5}=84$

$$\left[\left(\frac{1}{2}\right)^{x}\times\left(\frac{1}{2}\right)^{1}\right]+\left[\left(\frac{1}{2}\right)^{x}\times\left(\frac{1}{2}\right)^{3}\right]+\left[\left(\frac{1}{2}\right)^{x}\times\left(\frac{1}{2}\right)^{5}\right]=84$$

$$\left(\frac{1}{2}\right)^{x}\left[\frac{1}{2}+\frac{1}{8}+\frac{1}{32}\right]=84$$

$$\left(\frac{1}{2}\right)^{x}\times\frac{21}{32}=84$$

$$\left(\frac{1}{2}\right)^{x}=84\div\frac{21}{32}$$

$$\left(\frac{1}{2}\right)^{x}=128$$

128	2
64	2
32	2
16	2
8	2
4	2
2	2
1	

$$2^{-x}=2^{7}$$

$$\therefore -x=7 \Rightarrow \boxed{x=-7}$$

$$S.S.\ \text{in}\ \mathbb{R}=\{-7\}$$

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② **Find in $\mathbb{R}$ the solution set of each of the following equations :**

5 $2^{2x+1} - 33 \times 2^{x} + 16 = 0$

$2^{2x} \times 2^{1} - 33 \times 2^{x} + 16 = 0$

let $2^{x} = y$

$2^{2x} = y^{2}$

$2y^{2} - 33y + 16 = 0$

$y = 16$ $\qquad$ $y = \frac{1}{2}$

$2^{x} = 16$ $\qquad$ $2^{x} = \frac{1}{2}$

$2^{x} = 2^{4}$ $\qquad$ $2^{x} = 2^{-1}$

$\boxed{x = 4}$ $\qquad$ $\boxed{x = -1}$

S.S. in R = {−1, 4}

② **Find in ℝ the solution set of each of the following equations :**

6 $5^{2x-2} - 6 \times 5^{x-1} + 5 = 0$

$(5^{2x} \times 5^{-2}) - (6 \times 5^{x} \times 5^{-1}) + 5 = 0$

let $5^{x} = y$

$5^{2x} = y^2$

$\frac{1}{25} y^2 - \frac{6}{5} y + 5 = 0$

$y = 25$

$5^{x} = 5^{2}$

$\boxed{x = 2}$

$y = 5$

$5^{x} = 5^{1}$

$\boxed{x = 1}$

S.S. in $\mathbb{R} = \{1, 2\}$

math in use BY MICHAEL GAMIL

③ **Prove that :**

1 $$\frac{(27)^{y-\frac{1}{3}} \times \sqrt[y]{7^{y^2+3y}}}{(81)^{y-1} \times (21)^{5-y} \times (49)^{y-1}} = \frac{1}{9}$$

$$\text{L.H.S.} = \frac{(3^3)^{y-\frac{1}{3}} \times 7^{\frac{y^2+3y}{y}}}{(3^4)^{y-1} \times (3\times 7)^{5-y} \times (7^2)^{y-1}}$$

$$= \frac{(3)^{3y-1} \times 7^{y+3}}{(3)^{4y-4} \times 3^{5-y} \times 7^{5-y} \times 7^{2y-2}}$$

$$= 3^{\cancel{3y}-1-\cancel{4y}+4-5+\cancel{y}} \times 7^{\cancel{y}+\cancel{3}-\cancel{5}+\cancel{y}-\cancel{2y}+\cancel{2}}$$

$$= 3^{-2} \times 7^{0} = \left(\frac{1}{3}\right)^2 \times 1$$

$$= \frac{1}{9} = \text{R.H.S}$$

math in use

③ **Prove that :**

2 $$\frac{125 \times \sqrt[8]{4^3} \times 10^{-\frac{1}{4}}}{4^{\frac{5}{8}} \times \sqrt[4]{6^{-3}} \times (15)^{\frac{3}{4}}} = 25$$

$$L.H.S. = \frac{5^3 \times (2^2)^{\frac{3}{8}} \times (2 \times 5)^{-\frac{1}{4}}}{(2^2)^{\frac{5}{8}} \times (2 \times 3)^{-\frac{3}{4}} \times (3 \times 5)^{\frac{3}{4}}}$$

$$= \frac{5^3 \times 2^{\frac{3}{4}} \times 2^{-\frac{1}{4}} \times 5^{-\frac{1}{4}}}{2^{\frac{5}{4}} \times 2^{-\frac{3}{4}} \times 3^{-\frac{3}{4}} \times 3^{\frac{3}{4}} \times 5^{\frac{3}{4}}}$$

$$= 5^{3-\frac{1}{4}-\frac{3}{4}} \times 2^{\frac{3}{4}-\frac{1}{4}-\frac{5}{4}+\frac{3}{4}} \times 3^{\frac{3}{4}-\frac{3}{4}}$$

$$= 5^2 \times 2^0 \times 3^0$$

$$25 \times 1 \times 1 = 25 = R.H.S$$

4

1 If $y^{\frac{3}{4}} = 2x^{\frac{5}{3}} = 64$, then find the value of : $5x + 2y$

$\left(y^{\frac{3}{4}}\right)^{\frac{4}{3}} = (64)^{\frac{4}{3}}$

$2x^{\frac{5}{3}} = 64$

$\left(x^{\frac{5}{3}}\right)^{\frac{3}{5}} = (32)^{\frac{3}{5}}$

$y = 256$

$x = 8$

$5x + 2y = 5(8) + 2(256)$

$= 552$

4

2 If $x^{\frac{4}{3}} = 9\,y^{-\frac{2}{3}} = 81$, then find the value of : $|\,2\,x\,y\,|$

$x^{\frac{4}{3}} = 81$

$\left(x^{\frac{4}{3}}\right)^{\frac{3}{4}} = \pm(81)^{\frac{3}{4}}$

$\boxed{x = \pm 27}$

$9y^{-\frac{2}{3}} = 81$

$\left(y^{-\frac{2}{3}}\right)^{-\frac{3}{2}} = \pm(9)^{-\frac{3}{2}}$

$\boxed{y = \pm \frac{1}{27}}$

$|2xy| = |2 \times \pm \cancel{27} \times \pm \frac{1}{\cancel{27}}|$

$= |2| = 2$