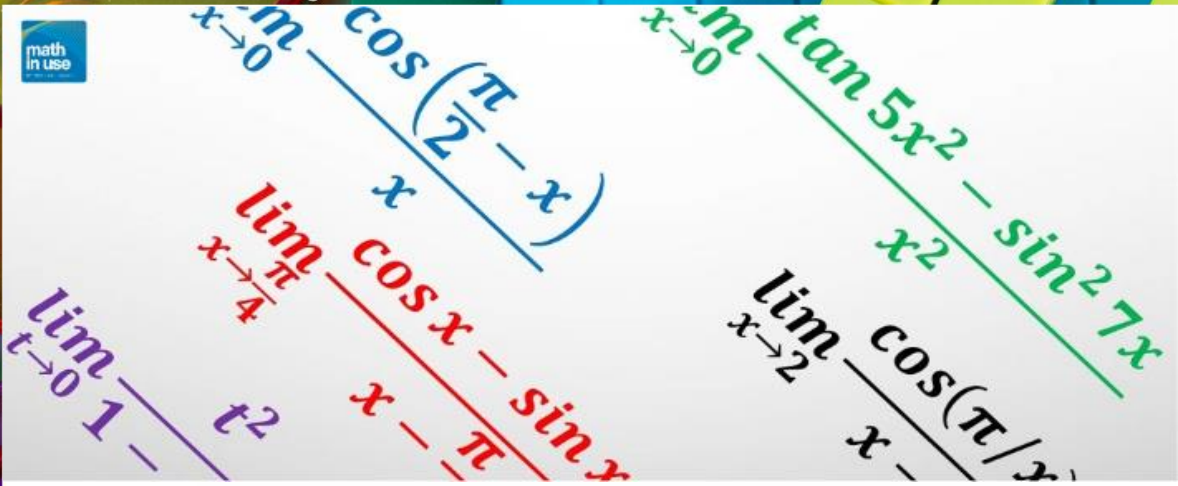

Exercise 8

Limits of trigonometric functions

Choose the correct answer

$\lim\limits_{x \to 0} \tan x =$ ……………

(a) zero (b) -1 (c) $\frac{\pi}{2}$ (d) π

$\lim\limits_{x \to \frac{\pi}{2}} \sin 2x = \dots\dots\dots\dots$

(a) zero (b) -1 (c) 1 (d) $\frac{\pi}{2}$

$\sin \cancel{2}\left(\frac{\pi}{\cancel{2}}\right) = \sin \pi = \text{zero}$

$\lim\limits_{x \to 0} \frac{2x}{\sin 3x} = \dots\dots\dots\dots$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 6 (d) has no existence.

$$\lim_{x \to 0} \frac{\frac{2\cancel{x}}{\cancel{x}}}{\frac{\sin 3x}{x}} = \frac{2}{3}$$

$\lim\limits_{x \longrightarrow 0} \frac{\tan \pi x}{6x} = \ldots\ldots \frac{\pi}{6} \ldots\ldots$

(a) π (b) $\frac{\pi}{6}$ (c) $\frac{1}{6}$ (d) has no existence.

$$\lim_{x \to 0} \frac{\sin\left(\frac{3}{5}\sqrt[4]{x}\right)}{\sqrt[4]{x}} = \dots\dots\dots\dots$$

(a) zero (b) $\frac{3}{5}$ (c) $\frac{5}{3}$ (d) $\sqrt[4]{\frac{3}{5}}$

$$\lim_{\sqrt[4]{x} \to 0} \frac{\sin\left(\frac{3}{5}\sqrt[4]{x}\right)}{\sqrt[4]{x}} = \frac{3}{5}$$

$$\lim_{(2x-3) \to 0} \frac{\tan(2x-3)}{2x-3} = \ldots\ldots 1 \ldots$$

(a) zero (b) -1 (c) 1 (d) does not exist.

$\lim_{h \to 0} \frac{\sin 3h^2}{4h^2} = \frac{3}{4}$

(a) zero (b) 1 (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

$$\lim_{x \to 0} \frac{3x^5}{\tan 4x^5} = \dots\dots\dots$$

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 1 (d) zero.

$$\lim_{x \to 0} \frac{\frac{3x^5}{x^5}}{\frac{\tan 4x^5}{x^5}} = \frac{3}{4}$$

$\lim\limits_{x \to 0} \dfrac{1 - \cos x}{3x} = \dots\dots\dots\dots$

(a) zero. (b) 1 (c) – 1 (d) does not exist.

$\frac{1}{3} \lim\limits_{x \to 0} \dfrac{1 - \cos x}{x} = \frac{1}{3}(0) = \text{Zero}$

$\lim\limits_{x \longrightarrow 0} \dfrac{4+5x}{\cos 3x} = \dfrac{4+(0)}{\cos 0} = \dfrac{4}{1} = 4$

(a) – 4 (b) 4 (c) 3 (d) – 3

$\lim\limits_{x \to 0} \frac{\sin \frac{x}{3}}{2x} = \lim\limits_{x \to 0} \frac{\sin \frac{1}{3}x}{2x} = \frac{\frac{1}{3}}{2} = \frac{1}{6}$

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

$\lim\limits_{x \to 0} \dfrac{2x + \sin 3x}{\tan 5x} = \ldots\ldots\ldots\ldots$

(a) 5 (b) $\frac{6}{5}$ (c) 1 (d) zero.

$$\lim_{x \to 0} \frac{\frac{2\cancel{x}}{\cancel{x}} + \frac{\sin 3x}{x}}{\frac{\tan 5x}{x}} = \frac{2+3}{5} = \underline{1}$$

$$\lim_{x \to 0} \frac{\sin 2x + 5 \sin 3x}{x} = \ldots\ldots\ldots\ldots\ldots$$

(a) 7 (b) 5 (c) 17 (d) 10

$$\lim_{x \to 0} \frac{\frac{\sin 2x}{x} + 5\frac{\sin 3x}{x}}{\frac{x}{x}} = \frac{2 + 5(3)}{1}$$

$$= 2 + 15 = 17$$

$\lim\limits_{x \to 0} \dfrac{2x + \sin 3x}{5x + \tan 2x} = \ldots\ldots\ldots\ldots$

(a) 1 (b) $\frac{5}{7}$ (c) $\frac{7}{5}$ (d) -1

$$\lim_{x \to 0} \frac{\frac{2\cancel{x}}{\cancel{x}} + \frac{\sin 3x}{x}}{\frac{5\cancel{x}}{\cancel{x}} + \frac{\tan 2x}{x}} = \frac{2+3}{5+2}$$

$$= \frac{5}{7}$$

$$\lim_{x \to 0} \frac{\sin 3x}{5x^2 - 4x} = \dots\dots\dots\dots$$

(a) $\frac{3}{4}$ (b) $\frac{-3}{4}$ (c) 3 (d) $\frac{3}{5}$

$$\lim_{x \to 0} \frac{\sin 3x}{x(5x-4)} = \lim_{x \to 0} \left(\frac{\sin 3x}{x} \times \frac{1}{5x-4} \right)$$

$$= 3 \times \frac{1}{-4} = -\frac{3}{4}$$

$\lim\limits_{x \to 0} \dfrac{\cancel{x} \sin 2x}{x^{\cancel{2}}} = \ldots\ldots\ldots\ldots\ldots$

(a) zero. (b) 1 (c) 2 (d) 4

$$\lim_{x \to 0} \frac{\sin 2x}{x} = 2$$

$\lim\limits_{x \to 0} \dfrac{\sin 2x \cos 3x}{6x} = \ldots\ldots\ldots\ldots\ldots$

(a) 1 (b) 3 (c) $\frac{1}{3}$ (d) zero.

$$\lim_{x \to 0} \left(\frac{\sin 2x}{6x} \times \cos 3x \right)$$

$$\frac{2}{6} \times 1 = \frac{1}{3}$$

$\lim\limits_{x \to 0} \dfrac{\sin 3x}{\tan \frac{1}{3} x} = \dots\dots\dots\dots$

(a) 9 (b) $\frac{1}{3}$ (c) 3 (d) 1

$$\lim_{x \to 0} \frac{\frac{\sin 3x}{x}}{\frac{\tan \frac{1}{3}x}{x}} = \frac{3}{\frac{1}{3}} = 9$$

$\lim\limits_{x \longrightarrow 0} \dfrac{\sin \frac{1}{2} x}{\sin \frac{3}{4} x} = \ldots\ldots\ldots\ldots\ldots$

(a) $\frac{1}{6}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

$$\lim_{x \to 0} \frac{\frac{\sin \frac{1}{2} x}{x}}{\frac{\sin \frac{3}{4} x}{x}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

$\lim\limits_{x \to 0} \dfrac{\tan^2 2x}{x \sin 3x} = \dots\dots\dots\dots$

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

$$\lim_{x\to 0} \frac{\dfrac{\tan^2 2x}{x^2}}{\dfrac{\cancel{x} \sin 3x}{x^{\cancel{2}}}} = \lim_{x\to 0} \frac{\left(\dfrac{\tan 2x}{x}\right)^2}{\dfrac{\sin 3x}{x}}$$

$$= \frac{4}{3}$$

$$\lim_{x \to 0} \frac{x^2 + \sin^2 2x}{x \tan 2x} = \dots\dots\dots\dots$$

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) 1 (d) 5

$$\lim_{x \to 0} \frac{\frac{\cancel{x^2}}{\cancel{x^2}} + \left(\frac{\sin 2x}{x}\right)^2}{\frac{\cancel{x} \tan x}{\cancel{x^2}}} = \frac{1 + (2)^2}{1} = 5$$

$\lim\limits_{x \to 0} \left(\frac{\sin 2x}{3x} + \frac{x \sin x^2}{\sin x^3} \right) = \ldots\ldots\ldots\ldots\ldots$

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

$$\lim_{x \to 0} \left(\frac{\sin 2x}{3x} + \frac{\frac{\cancel{x} \sin x^2}{x^{\cancel{3}\,2}}}{\frac{\sin x^3}{x^3}} \right)$$

$$\frac{2}{3} + \frac{1}{1} = \frac{5}{3} = 1\frac{2}{3}$$

$$\lim_{x \to 0} \frac{\sin^2 9x + \tan 16x^2}{4x^2} = \ldots\ldots\ldots\ldots\ldots$$

(a) $\frac{97}{4}$ (b) $\frac{97}{16}$ (c) $\frac{9}{4}$ (d) 4

$$\lim_{x \to 0} \frac{\left(\frac{\sin 9x}{x}\right)^2 + \frac{\tan 16x^2}{x^2}}{\frac{4x^2}{x^2}} = \frac{(9)^2 + 16}{4}$$

$$= \frac{97}{4}$$

$\lim\limits_{x \to 0} 3x \csc 2x =$ ....

(a) 6 (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) has no existence.

$$\lim_{x \to 0} \frac{\frac{3x}{x}}{\frac{\sin 2x}{x}} = \frac{3}{2}$$

$\lim\limits_{x \longrightarrow 0} \dfrac{\sin 2x \tan 3x}{4x^2} = \ldots\ldots\ldots\ldots\ldots$

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) 6

$\frac{1}{4} \lim\limits_{x \to 0} \left(\frac{\sin 2x}{x} \times \frac{\tan 3x}{x} \right)$

$\frac{1}{4} (2 \times 3) = \frac{6}{4} = \frac{3}{2}$

$\lim_{x \to 0} \frac{1 - \tan x}{\sin x - \cos x} = \frac{1-0}{0-1} = \frac{1}{-1} = -1$

(a) 1 (b) – 1 (c) zero. (d) has no existence.

$$\lim_{x \to 0} \frac{12 - 12 \cos x}{x} = \dots\dots\dots\dots\dots$$

(a) zero (b) 12 (c) 1 (d) 24

$$\lim_{x \to 0} \frac{12(1 - \cos x)}{x} = 12(0)$$

$$= \text{Zero}$$

$$\lim_{x \longrightarrow 0} \frac{\sin x - \sin x \cos x}{x^2} = \ldots\ldots\ldots\ldots\ldots$$

(a) zero (b) 1 (c) 2 (d) 3

$$\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{x^2}$$

$$\lim_{x \to 0} \left(\frac{\sin x}{x} \times \frac{1 - \cos x}{x} \right)$$

$1 \times$ zero $=$ zero

$\lim\limits_{x \to 0} \dfrac{\cos x + 2x - 1}{3x} = \dots\dots\dots\dots$

(a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) 0

$\lim\limits_{x \to 0} \left(\dfrac{\cos x - 1}{3x} + \dfrac{2\cancel{x}}{3\cancel{x}} \right)$

$\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$

$= \text{zero} + \dfrac{2}{3} = \dfrac{2}{3}$

$$\lim_{x \to 0} \frac{1 - \cos x + \sin 3x}{1 - \cos x + \tan 2x} = \dots\dots\dots\dots$$

(a) zero (b) $\frac{3}{2}$ (c) 1 (d) $\frac{4}{3}$

$$\lim_{x \to 0} \frac{\frac{1-\cos x}{x} + \frac{\sin 3x}{x}}{\frac{1-\cos x}{x} + \frac{\tan 2x}{x}} = \frac{0+3}{0+2} = \frac{3}{2}$$

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$\lim_{x \to 0} \frac{1 - \sec x}{\cos x - 1} = \ldots\ldots\ldots\ldots$

(a) 2 (b) 1 (c) 0 (d) – 1

$\lim_{x\to 0} \frac{1-\sec x}{\cos x - 1} \times \frac{1+\sec x}{1+\sec x}$

$1+\tan^2 x = \sec^2 x$

$\boxed{1-\sec^2 x = -\tan^2 x}$

$\sin^2 x + \cos^2 x = 1$

$1-\cos^2 x = \sin^2 x$

$\lim_{x\to 0} \frac{1-\sec^2 x}{\cos x + \cancel{1} - \cancel{1} - \sec x}$

$\lim_{x\to 0} \frac{-\tan^2 x}{\frac{\cos x}{1} - \frac{1}{\cos x}} = \lim_{x\to 0} \frac{-\tan^2 x}{\frac{\cos^2 x - 1}{\cos x}}$

$\lim_{x\to 0} \frac{-\cos x \cdot \tan^2 x}{\sin^2 x} = \lim_{x\to 0} \frac{-\cancel{\cos x} \cdot \frac{\cancel{\sin^2 x}}{\cos^{\cancel{2}} x}}{\cancel{\sin^2 x}}$

$= \lim_{x\to 0} -\frac{1}{\cos x} = \lim_{x\to 0} -\sec x$

$= -1$

If $\lim\limits_{x \to 0} \frac{(a+3)\,x}{\sin a\,x} = \frac{2}{5}$, then a = ...

(a) – 5 (b) – 3 (c) – 1 (d) 3

$$\frac{a+3}{a} = \frac{2}{5}$$

$$5a + 15 = 2a$$

$$5a - 2a = -15$$

$$3a = -15 \Rightarrow \boxed{a = -5}$$

If $\lim\limits_{x \to 0} \frac{\sin^2 a x - \tan 2 a x^2}{x^2} = -1$, then a = ……………

(a) – 1 (b) 2 (c) 1 (d) 4

$$\lim_{x\to 0}\left(\frac{\sin ax}{x}\right)^2 - \lim_{x\to 0}\left(\frac{\tan 2ax^2}{x^2}\right) = -1$$

$$a^2 - 2a = -1$$

$$a^2 - 2a + 1 = 0$$

$$\boxed{a = 1}$$

If $\lim\limits_{x \to 0} \frac{\sin ax}{b x} = \frac{1}{3}$, $\lim\limits_{x \to 0} \frac{\tan bx}{c x} = \frac{4}{3}$, then $\frac{a}{c} = \dots\dots\dots\dots$

(a) -4 (b) $\frac{1}{4}$ (c) $\frac{4}{9}$ (d) $\frac{1}{12}$

$\frac{a}{b} = \frac{1}{3}$ $\frac{b}{c} = \frac{4}{3}$

$a : b : c$

$1 : 3$

$4 : 3$

$4 : 12 : 9 \Rightarrow \frac{a}{c} = \frac{4}{9}$

If $\lim\limits_{x \to 0} \frac{\sin a\cancel{x}}{\sin b\cancel{x}} = \frac{3}{5}$, then $\frac{a+b}{a-b} = \dots\dots\dots\dots\dots$

(a) – 4 (b) 8 (c) 2 (d) 4

$$\frac{a}{b} = \frac{3}{5} \Rightarrow \begin{matrix} a = 3m \\ b = 5m \end{matrix}$$

$$\frac{a+b}{a-b} = \frac{3m+5m}{3m-5m} = \frac{8\cancel{m}}{-2\cancel{m}} = -4$$

$$\lim_{x \to 0} \frac{(\sin x + \cos x)^2 - 1}{3x} = \dots\dots\dots\dots$$

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) zero.

$$\lim_{x \to 0} \frac{\cancel{\sin^2 x + \cos^2 x} + 2\sin x\cos x \cancel{-1}}{3x}$$

$$\lim_{x \to 0} \frac{2\sin x}{3x} \cdot \cos x = \frac{2}{3}(1)(1) = \frac{2}{3}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin^3 x}{\cos^2 x} = \ldots\ldots\ldots\ldots\ldots$$

(a) 1 (b) $\frac{3}{2}$ (c) $\frac{3}{4}\pi$ (d) zero.

$1 - y^3 = (1-y)(1+y+y^2)$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$$\lim_{x \to \frac{\pi}{2}} \frac{\cancel{(1-\sin x)}(1+\sin x+\sin^2 x)}{\cancel{(1-\sin x)}(1+\sin x)}$$

$$= \frac{1+1+(1)^2}{1+1} = \frac{3}{2}$$

$$\lim_{x \to 1} \frac{\tan(x-1)}{\sqrt{x}-1} = \dots\dots\dots\dots$$

(a) 2 (b) zero. (c) -2 (d) $\frac{1}{2}$

$$\lim_{x \to 1} \frac{\frac{\tan(x-1)}{x-1}}{\frac{\sqrt{x}-1}{x-1}}$$

$$\lim_{x \to 1} \frac{\tan(x-1)}{x-1} \div \lim_{x \to 1} \frac{\sqrt{x}-1}{x-1}$$

$$\lim_{(x-1) \to 0} \frac{\tan(x-1)}{x-1} \times \lim_{x \to 1} \frac{x^1 - 1^1}{x^{\frac{1}{2}} - 1^{\frac{1}{2}}}$$

$$1 \times \frac{1}{\frac{1}{2}} (1)^{1-\frac{1}{2}}$$

$$1 \times 2 \times 1 = 2$$

$$\lim_{x \to 1} \frac{\sin(x-1)}{x^2-1} = \ldots\ldots\ldots\ldots$$

(a) $\frac{1}{2}$ (b) 1 (c) -1 (d) zero.

$$\lim_{x \to 1} \frac{\frac{\sin(x-1)}{x-1}}{\frac{x^2-1}{x-1}}$$

$$\lim_{x \to 1} \frac{\sin(x-1)}{x-1} \times \lim_{x \to 1} \frac{x-1}{x^2-1}$$

$$\lim_{(x-1) \to 0} \frac{\sin(x-1)}{x-1} \times \lim_{x \to 1} \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}}$$

$$1 \times \frac{1}{2} = \frac{1}{2}$$

$$\lim_{x \to 3} \frac{\tan(x-3)}{x^2-x-6} = \dots\dots\dots$$

(a) $\frac{1}{5}$ (b) $\frac{8}{3}$ (c) 3 (d) – 3

$$\lim_{x \to 3} \frac{\tan(x-3)}{(x-3)(x+2)}$$

$$\lim_{(x-3) \to 0} \frac{\tan(x-3)}{x-3} \times \lim_{x \to 3} \frac{1}{x+2}$$

$$1 \times \frac{1}{5} = \frac{1}{5}$$

$\lim\limits_{x \to \frac{\pi}{2}} \frac{3\cos x}{\pi - 2x} = \ldots\ldots\ldots\ldots$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$

(a) 3 (b) $\frac{3}{2}$ (c) $\frac{-3}{2}$ (d) -3

$x \to \frac{\pi}{2}$

$x - \frac{\pi}{2} \to 0$

$\left(\frac{\pi}{2} - x\right) \to 0$

$$\lim_{\left(\frac{\pi}{2}-x\right) \to 0} \frac{3\sin\left(\frac{\pi}{2}-x\right)}{2\left(\frac{\pi}{2}-x\right)}$$

$$\frac{3}{2} \times 1 = \frac{3}{2}$$

$\lim\limits_{x \to \pi} \dfrac{\sin x}{\pi - x} = \dots\dots\dots\dots$

$\sin(\pi - x) = \sin x$

(a) 1 (b) π^2 (c) π (d) $-\pi$

$\lim\limits_{(\pi - x) \to 0} \dfrac{\sin(\pi - x)}{\pi - x} = 1$

$\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2} = \ldots\ldots\ldots\ldots\ldots$

(a) 1 (b) does not exist. (c) 0 (d) $\frac{1}{2}$

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} \times \frac{1 + \cos x}{1 + \cos x}$$

$$\lim_{x \to 0} \frac{1 - \cos^2 x}{x^2 (1 + \cos x)} = \lim_{x \to 0} \frac{\sin^2 x}{x^2 (1 + \cos x)}$$

$$\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2 \times \lim_{x \to 0} \frac{1}{1 + \cos x}$$

$$(1)^2 \times \frac{1}{1+1} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\sin(\sin x)}{3x} = \dots\dots\dots\dots$$

(a) zero. (b) 1 (c) $\frac{1}{3}$ (d) 3

$$\lim_{\sin x \to 0} \frac{\frac{\sin(\sin x)}{\sin x}}{\frac{3x}{\sin x}} = \frac{1}{3}$$

$\lim\limits_{x \to 0} \frac{\sin x}{x} = \dots\dots\dots\dots$, where x is in degrees.

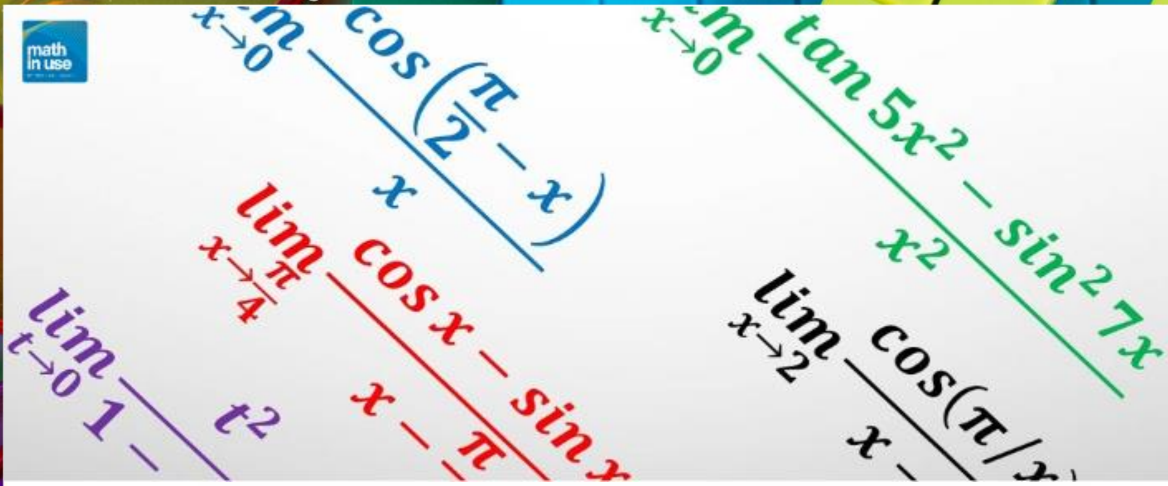
(a) 1 (b) $\frac{\pi}{180}$ (c) $\frac{180}{\pi}$ (d) π

$\lim\limits_{x \to 0} \frac{\sin x \times \frac{\pi}{180}}{x}$

$= \lim\limits_{x \to 0} \frac{\sin \frac{\pi}{180} x}{x} = \frac{\pi}{180}$

$\frac{x^{\circ}}{180} = \frac{x^{\text{rad}}}{\pi}$

$x^{\text{rad}} = x^{\circ} \times \frac{\pi}{180}$



Exercise 8

Limits of trigonometric functions

① **Find each of the following**

1. $\lim\limits_{x \to 0} \dfrac{3x + \sin x}{2x + \tan 3x}$ $= \lim\limits_{x \to 0} \dfrac{\frac{3x}{x} + \frac{\sin x}{x}}{\frac{2x}{x} + \frac{\tan 3x}{x}}$

$$= \frac{3 + 1}{2 + 3} = \frac{4}{5}$$

2 $\lim\limits_{x \to 0} \frac{\sin x - 3\tan x}{5x\cos x} = \lim\limits_{x \to 0} \frac{\frac{\sin x}{x} - \frac{3\tan x}{x}}{\frac{5\cancel{x}\cos x}{\cancel{x}}}$

$= \frac{1 - 3(1)}{5(1)} = \frac{-2}{5}$

3 $\lim_{x \to 0} \frac{x + x\cos x}{\sin x \cos x} = \lim_{x \to 0} \frac{\frac{\cancel{x}}{\cancel{x}} + \frac{\cancel{x}\cos x}{\cancel{x}}}{\frac{\sin x \cos x}{x}}$

$= \frac{1+1}{(1)(1)} = \frac{2}{1} = 2$

4 $\lim\limits_{x \to 0} \dfrac{x^2 + \sin 3x}{5x \cos 2x}$ $= \lim\limits_{x \to 0} \dfrac{\dfrac{x^{\cancel{2}}}{\cancel{x}} + \dfrac{\sin 3x}{x}}{\dfrac{5\cancel{x} \cos 2x}{\cancel{x}}}$

$= \dfrac{0 + 3}{5(1)} = \dfrac{3}{5}$

5 $\lim\limits_{x \longrightarrow 0} \frac{2}{x} \sin \frac{x}{7}$ $= \lim\limits_{x \to 0} \frac{2 \sin \frac{1}{7} x}{x}$

$= 2 \left(\frac{1}{7} \right) = \frac{2}{7}$

6 $\lim\limits_{x \to 0} \left(\frac{3}{5x} + 4 \right) \sin 4x$

$$\lim_{x \to 0} \left(\frac{3 \sin 4x}{5x} + 4 \sin 4x \right)$$

$$\frac{3}{5} \times 4 + \cancel{4(0)} = \frac{12}{5}$$

7 $\lim\limits_{x \to 0} \dfrac{\sin 24x \times \cos 6x}{\tan 6x \times \cos 24x} = \lim\limits_{x \to 0} \dfrac{\dfrac{\sin 24x \cos 6x}{x}}{\dfrac{\tan 6x \times \cos 24}{x}}$

$= \dfrac{24 \times 1}{6 \times 1} = \dfrac{24}{6} = 4$

8 $\lim\limits_{x \longrightarrow 0} \dfrac{\sin^2 x}{x} = \lim\limits_{x \to 0} \dfrac{\sin x \cdot \sin x}{x}$

$= \lim\limits_{x \to 0} \left(\dfrac{\sin x}{x} \times \sin x \right)$

$1 \times 0 = \text{zero}$

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9 $\lim\limits_{x\to 0} \frac{\sin^2 3x}{5x^2}$ $= \cancel{\lim\limits_{x\to 0}\left(\frac{\sin 3x}{5x}\right)^2 = \boxed{\frac{9}{25}}}$

$\frac{1}{5}\lim\limits_{x\to 0}\left(\frac{\sin 3x}{x}\right)^2$

$\frac{1}{5}(3)^2 = \frac{9}{5}$

$\boxed{10}$ $\lim\limits_{x \to 0} \dfrac{\sin^2 \dfrac{x}{2}}{x \tan 2x}$ $= \lim\limits_{x \to 0} \dfrac{\left(\dfrac{\sin \frac{1}{2}x}{x}\right)^2}{\dfrac{\cancel{x} \tan 2x}{x\cancel{^2}}}$

$= \dfrac{\left(\frac{1}{2}\right)^2}{2} = \dfrac{\frac{1}{4}}{2} = \dfrac{1}{8}$

11 $\lim\limits_{x \to 0} \dfrac{\tan^3 2x}{4x^3} = \dfrac{1}{4} \lim\limits_{x \to 0} \left(\dfrac{\tan 2x}{x}\right)^3$

$= \dfrac{1}{4}(2)^3 = \dfrac{8}{4} = 2$

$\boxed{12}$ $\lim\limits_{x \to 0} \dfrac{\sin 4x \tan^2 5x}{x^2 \sin x} = \lim\limits_{x \to 0} \dfrac{\dfrac{\sin 4x}{x} \times \left(\dfrac{\tan 5x}{x}\right)^2}{\cancel{\dfrac{x^3}{x^3}} \times \dfrac{\sin x}{x}}$

$= \dfrac{4 \times (5)^2}{1 \times 1} = 4 \times 25 = 100$

13 $\lim_{x \to 0} \frac{x^2 + \tan^2 2x}{2x^2 + \sin^2 3x} = \lim_{x \to 0} \frac{\frac{x^2}{x^2} + \frac{\tan^2 2x}{x^2}}{\frac{2\cancel{x^2}}{\cancel{x^2}} + \frac{\sin^2 3x}{x^2}}$

$\lim_{x \to 0} \frac{1 + \left(\frac{\tan 2x}{x}\right)^2}{2 + \left(\frac{\sin 3x}{x}\right)^2} = \frac{1 + (2)^2}{2 + (3)^2}$

$= \frac{1+4}{2+9} = \frac{5}{11}$

14 $\lim\limits_{x \to 0} \dfrac{2x^3 + x\sin 5x}{x^2 - \tan 3x^2} = \lim\limits_{x \to 0} \dfrac{\dfrac{2\cancel{x^3}}{\cancel{x^2}} + \dfrac{\cancel{x}\sin 5x}{x^{\cancel{2}}}}{\dfrac{\cancel{x^2}}{\cancel{x^2}} - \dfrac{\tan 3x^2}{x^2}}$

$$\lim_{x \to 0} \frac{2x + \dfrac{\sin 5x}{x}}{1 - \dfrac{\tan 3x^2}{x^2}} = \frac{0 + 5}{1 - 3}$$

$$= \frac{5}{-2} = -\frac{5}{2}$$

15 $\lim\limits_{x \to 0} \dfrac{x \sin 2x + \sin^2 2x}{\tan^2 3x + x^2} = \lim\limits_{x \to 0} \dfrac{\dfrac{\cancel{x} \sin 2x}{x^{\cancel{2}}} + \dfrac{\sin^2 2x}{x^2}}{\dfrac{\tan^2 3x}{x^2} + \dfrac{\cancel{x^2}}{\cancel{x^2}}}$

$$= \lim_{x \to 0} \frac{\dfrac{\sin 2x}{x} + \left(\dfrac{\sin 2x}{x}\right)^2}{\left(\dfrac{\tan 3x}{x}\right)^2 + 1}$$

$$= \frac{2 + (2)^2}{(3)^2 + 1} = \frac{6}{10} = \frac{3}{5}$$

16 $\lim\limits_{x \to 0} \frac{\sin 5x^3 + \sin^3 5x}{2x^3} = \lim\limits_{x \to 0} \frac{\frac{\sin 5x^3}{x^3} + \left(\frac{\sin 5x}{x}\right)^3}{\frac{2\cancel{x^3}}{\cancel{x^3}}}$

$= \frac{5 + (5)^3}{2} = \frac{5 + 125}{2} = 65$

17 $\lim\limits_{x \longrightarrow 0} \frac{\tan^2 x + \tan^2 3x + \tan^2 5x}{x^2}$

$$\lim_{x \to 0} \left[\left(\frac{\tan x}{x} \right)^2 + \left(\frac{\tan 3x}{x} \right)^2 + \left(\frac{\tan 5x}{x} \right)^2 \right]$$

$$= (1)^2 + (3)^2 + (5)^2$$

$$= 1 + 9 + 25 = 35$$

18 $\lim\limits_{x \to 0} \dfrac{\tan 6x - \sin(-3x)}{x(\cos 5x + \cos 2x)}$

$$\lim_{x \to 0} \frac{\boxed{\dfrac{\tan 6x}{x}} + \boxed{\dfrac{\sin 3x}{x}}}{\dfrac{\cancel{x}(\cos 5x + \cos 2x)}{\cancel{x}}} = \frac{6+3}{1+1}$$

$$= \frac{9}{2}$$

19 $\lim\limits_{x \to 3} \frac{\tan(x-3)}{x^3-27}$ $= \lim\limits_{x \to 3} \frac{\tan(x-3)}{(x-3)(x^2+3x+9)}$

$$\lim_{(x-3) \to 0} \frac{\tan(x-3)}{x-3} \times \lim_{x \to 3} \frac{1}{x^2+3x+9}$$

$$= 1 \times \frac{1}{9+9+9} = \frac{1}{27}$$

20 $\lim\limits_{x \to 0} \dfrac{3(1-\cos x)}{x}$ $= 3 \lim\limits_{x \to 0} \dfrac{1-\cos x}{x}$

$= 3(0) = \text{Zero}$

21 $\lim\limits_{x \to 0} \dfrac{1-\cos x+\sin x}{1-\cos x-\sin x}$ $= \lim\limits_{x \to 0} \dfrac{\frac{1-\cos x}{x}+\frac{\sin x}{x}}{\frac{1-\cos x}{x}-\frac{\sin x}{x}}$

$= \dfrac{0+1}{0-1} = -1$

22 $\lim_{x \to 0} \frac{\sin x (1 - \cos x)}{x^2}$

$\lim_{x \to 0} \left(\frac{\sin x}{x} \times \frac{1 - \cos x}{x} \right)$

$= 1 \times 0 = \text{Zero}$

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 - \cos^2\theta = \sin^2\theta$$

23 $\lim\limits_{x \to 0} \dfrac{1 - \cos^2 3x}{4x^2}$

$$\lim_{x \to 0} \frac{\sin^2 3x}{4x^2} = \frac{1}{4}\lim_{x \to 0}\left(\frac{\sin 3x}{x}\right)^2$$

$$= \frac{1}{4}(3)^2 = \frac{9}{4}$$

24 $\lim\limits_{x \to 0} \dfrac{3 - 3\cos^2 4x}{8x^2} = \lim\limits_{x \to 0} \dfrac{3(1-\cos^2 4x)}{8x^2}$

$= \lim\limits_{x \to 0} \dfrac{3\sin^2 4x}{8x^2} = \dfrac{3}{8} \lim\limits_{x \to 0} \left(\dfrac{\sin 4x}{x}\right)^2$

$= \dfrac{3}{8}(4)^2 = \dfrac{3}{8} \times 16 = 6$

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25 $\lim\limits_{x\to 0} \frac{1-\cos x}{x^2} \times \frac{1+\cos x}{1+\cos x}$

$$\lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)}$$

$$\lim_{x\to 0} \frac{\sin^2 x}{x^2(1+\cos x)}$$

$$\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 \times \lim_{x\to 0} \frac{1}{1+\cos x}$$

$$(1)^2 \times \frac{1}{1+1} = \frac{1}{2}$$

26 $\lim_{x \to 0} \cot 3x \sin 5x$

$$= \lim_{x \to 0} \frac{\frac{\sin 5x}{x}}{\frac{\tan 3x}{x}}$$

$$= \frac{5}{3}$$

27 $\lim_{x \to 0} 3x \tan 2x \csc^2 3x$

$$\lim_{x \to 0} \frac{\frac{3\cancel{x} \tan 2x}{x^{\cancel{2}}}}{\frac{\sin^2 3x}{x^2}} = \lim_{x \to 0} \frac{\frac{3 \tan 2x}{x}}{\left(\frac{\sin 3x}{x}\right)^2}$$

$$= \frac{\cancel{3} \times 2}{(3)^{\cancel{2}}} = \frac{2}{3}$$

28 $\lim\limits_{x \longrightarrow 0} \csc 4x \cos\left(\frac{\pi}{2} - x\right)$

$$\lim_{x \to 0} \frac{\frac{\sin x}{x}}{\frac{\sin 4x}{x}} = \frac{1}{4}$$

29 $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sin\left(2x - \frac{\pi}{2}\right)}{\tan(4x - \pi)}$

$= \lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sin\left[-\left(\frac{\pi}{2} - 2x\right)\right]}{\tan\left[-(\pi - 4x)\right]}$

$x \to \frac{\pi}{4}$

$2x \to \frac{\pi}{2}$

$4x \to \pi$

$\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sin\left(\frac{\pi}{2} - 2x\right)}{\tan(\pi - 4x)}$

$\lim \dfrac{\dfrac{\sin\left(\frac{\pi}{2} - 2x\right)}{\pi/2 - 2x}}{\dfrac{\tan(\pi - 4x)}{\pi - 4x}} = \dfrac{1}{1} = 1$

30 $\lim\limits_{x \longrightarrow 0} \dfrac{\cos\left(\frac{\pi}{2} - x\right)}{\cot\left(\frac{\pi}{2} - 3x\right)} = \lim\limits_{x \to 0} \dfrac{\frac{\sin x}{x}}{\frac{\tan 3x}{x}} = \dfrac{1}{3}$

$\sin(\pi - x) = \sin x$

31 $\displaystyle\lim_{(x-\pi)\to 0} \frac{\sin x}{x-\pi}$

$$\lim_{\pi - x \to 0} \frac{\sin(\pi - x)}{-(\pi - x)} = -1$$

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32 $\lim\limits_{x \to \frac{\pi}{2}} \frac{2x-\pi}{\cos x}$

$\boxed{\sin\left(\frac{\pi}{2}-x\right) = \cos x}$

$\sin\left(\frac{\pi}{2}+x\right) = \cos x$

$\lim\limits_{\left(\frac{\pi}{2}-x\right) \to 0} \frac{-2\left(\frac{\pi}{2}-x\right)}{\sin 1\left(\frac{\pi}{2}-x\right)} = \frac{-2}{1} = -2$

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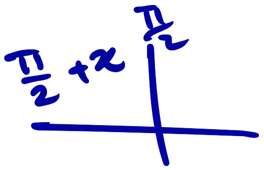
$\sin x = \sin(\pi - x)$

$\boxed{33}$ $\lim_{x \to 1} \frac{\sin x\pi}{1 - x}$

$$\lim_{(x-1) \to 0} \frac{\sin(\pi - x\pi)}{1 - x} = \lim_{(1-x) \to 0} \frac{\sin \pi(1-x)}{(1-x)}$$

$$= \pi$$



34 $\lim\limits_{x \to -1} \dfrac{1+x}{\cos \frac{\pi}{2} x}$

$$\lim_{(x+1)\to 0} \frac{1+x}{\sin\left(\frac{\pi}{2}+\frac{\pi}{2}x\right)} = \lim_{(1+x)\to 0} \frac{(1+x)}{\sin \frac{\pi}{2}(1+x)}$$

$$= \frac{1}{\pi/2} = \frac{2}{\pi}$$