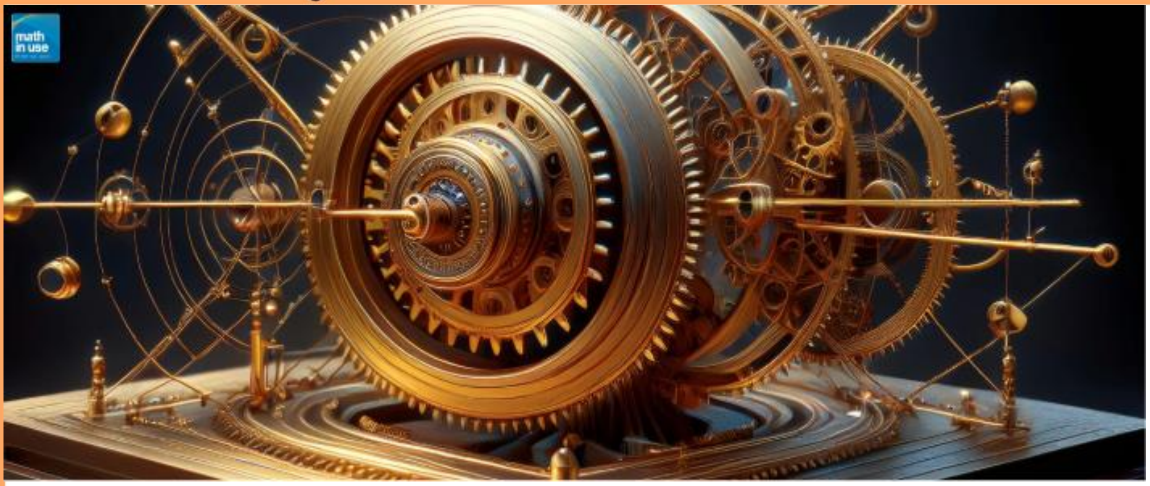


Exercise 3

The resultant of coplanar forces meeting at a point (part 1)

Choose the correct answer

If $\overrightarrow{F_1} = \overrightarrow{i} - \overrightarrow{j}$, $\overrightarrow{F_2} = 2\overrightarrow{i} - 4\overrightarrow{j}$, $\overrightarrow{R} = 2a\overrightarrow{i} - 3b\overrightarrow{j}$, then $a + b = \cdots\cdots\cdots$

(a) 3 (b) $3\frac{1}{3}$ (c) $3\frac{1}{6}$ (d) 12

$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

$(2a, -3b) = (1, -1) + (2, -4)$

$2a = 3$ $\qquad$ $-3b = -5$

$a = \frac{3}{2}$ $\qquad$ $b = \frac{5}{3}$

$a + b = \frac{3}{2} + \frac{5}{3} = \frac{19}{6} = 3\frac{1}{6}$

If $\overrightarrow{F_1} = 3\,\vec{i} - 2\,\vec{j}$, $\overrightarrow{F_2} = a\,\vec{i} - \vec{j}$, $\overrightarrow{F_3} = 4\,\vec{i} - b\,\vec{j}$, $\vec{R} = 6\,\vec{i} - 4\,\vec{j}$

, then (a , b) = ...

(a) (1 , – 1) (b) (– 1 , 1) (c) (– 1 , – 1) (d) (1 , 1)

$\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = R$

$(3,-2) + (a,-1) + (4,-b) = (6,-4)$

$a + 7 = 6$ $-3 - b = -4$

$a = -1$ $-b = -1$

$b = 1$

$(a, b) = (-1, 1)$

If $\overrightarrow{F_1} = 4\,\vec{i}$, $\overrightarrow{F_2} = 8\,\vec{i} - 5\,\vec{j}$, then $\|\vec{R}\| =$ .... force unit.

(a) 12 (b) 5 (c) 13 (d) $\sqrt{73}$

$$\vec{R} = \overrightarrow{F_1} + \overrightarrow{F_2} = (4,0) + (8,-5)$$

$$= (12,-5)$$

$$\|\vec{R}\| = \sqrt{(12)^2 + (-5)^2} = 13 \text{ F.U.}$$

If $\overrightarrow{F_1} = 3\,\vec{i} + 2\,\vec{j}$, $\overrightarrow{F_2} = a\,\vec{i} + 7\,\vec{j}$, $\overrightarrow{F_3} = -12\,\vec{i} + b\,\vec{j}$ are three coplanar forces meeting at a point and the resultant $\vec{R} = \left(6\sqrt{2}\,,\,\frac{3}{4}\pi\right)$, then $a - b =$ ....

(a) -3 (b) 3 (c) zero (d) 6

$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

$$(3,2) + (a,7) + (-12,b) = \left(6\sqrt{2}\cos\frac{3\pi}{4}, 6\sqrt{2}\sin\frac{3\pi}{4}\right)$$

$$(a-9\,,\,b+9) = (-6\,,6)$$

$$a - 9 = -6 \qquad b + 9 = 6$$

$$a = 3 \qquad b = -3$$

$$a - b = 3 - (-3) = 6$$

Three coplanar forces $\overrightarrow{F_1} = 6\,\overrightarrow{i} + 7\,\overrightarrow{j}$, $\overrightarrow{F_2} = a\,\overrightarrow{i} - 9\,\overrightarrow{j}$, $\overrightarrow{F_3} = 5\,\overrightarrow{i} + b\,\overrightarrow{j}$ act at a particle and they are in equilibrium , then a + 2 b = ...

(a) – 9 (b) 5 (c) 7 (d) – 7

$$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3}$$

$$(6,7) + (a,-9) + (5,b) = (0,0)$$

$a + 11 = 0$ $\qquad$ $b - 2 = 0$

$a = -11$ $\qquad$ $b = 2$

$$a + 2b = -11 + 4 = -7$$

If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ and $\overrightarrow{F_3}$ are three coplanar equilibrium forces meeting at a point ,
and $\overrightarrow{F_1} = 2\,\vec{i} - 3\,\vec{j}$, $\overrightarrow{F_2} = 3\,\vec{i} + 5\,\vec{j}$, then $\overrightarrow{F_3} =$ ....

(a) $-5\,\vec{i} - 2\,\vec{j}$ (b) $-5\,\vec{i} + 2\,\vec{j}$ (c) $5\,\vec{i} + 2\,\vec{j}$ (d) $5\,\vec{i} - 2\,\vec{j}$

$$\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = \vec{0}$$

$$(2,-3) + (3,5) + \overrightarrow{F_3} = \vec{0}$$

$$\overrightarrow{F_3} + (5,2) = \vec{0}$$

$$\overrightarrow{F_3} = \vec{0} - (5,2) = (-5,-2)$$

$$\overrightarrow{F_3} = -5\vec{i} - 2\vec{j}$$

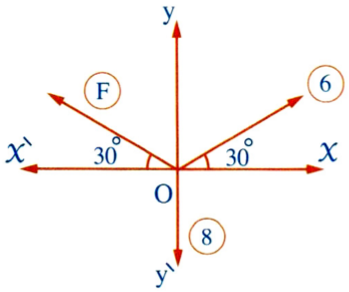
If the resultant of the forces in the given figure acts in direction of y-axis , then F = ... force unit.

(a) 2

(b) 6

(c) 8

(d) 14



$(6, 30^\circ)$, $(F, 150^\circ)$, $(8, 270^\circ)$

$\because$ Resultant act in y-axis $\Rightarrow \therefore \boxed{x = 0}$

$X = 6\cos 30 + F\cos 150^\circ + 8\cos 270 = 0$

$3\sqrt{3} - \frac{\sqrt{3}}{2}F = 0$

$\therefore 3\sqrt{3} = \frac{\sqrt{3}}{2}F$

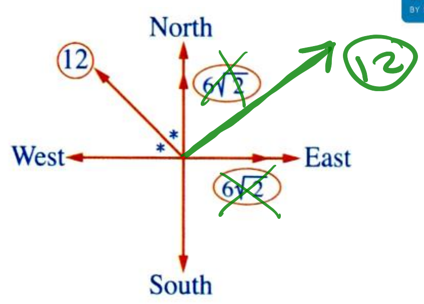
$\therefore F = 6$

In the opposite figure :

The direction of the resultant of the forces is ....

(a) South. (b) East.

(c) West. (d) North.



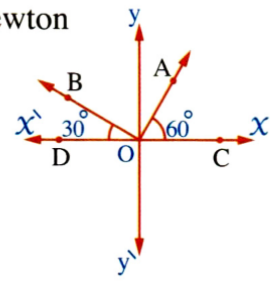
$6\sqrt{2}$ & $6\sqrt{2}$ " two equal forces their resultant will act in eastern north direction & will equal 12

$2F\cos\frac{\alpha}{2} = 2(6\sqrt{2})\cos 45 = 12$

12 & 12 " two equal forces their resultant will act in North direction

In the opposite figure :

The magnitude of four coplanar forces are 1 , 2 , $4\sqrt{3}$, $3\sqrt{3}$ newton act at point O in the direction of $\overrightarrow{OX}$, $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OY}$, m (∠ AOC) = 60° , m (∠ BOD) = 30° , then the magnitude and the direction of the resultant of the forces is ....



(a) (4 , 180°)   (b) (4 , 0°)

(c) (3 , 0°)   (d) (5 , 90°)

$(1, 0^\circ)$, $(2, 60^\circ)$, $(4\sqrt{3}, 150^\circ)$, $(3\sqrt{3}, 270)$

$X = 1\cos 0 + 2\cos 60 + 4\sqrt{3}\cos 150^\circ + 3\sqrt{3}\cos 270 = -4$

$y = 1\sin 0 + 2\sin 60 + 4\sqrt{3}\sin 150 + 3\sqrt{3}\sin 270 = 0$

$\overrightarrow{R} = (-4, 0) \Rightarrow \|\overrightarrow{R}\| = 4$ & acts in -ve direction of x-axis

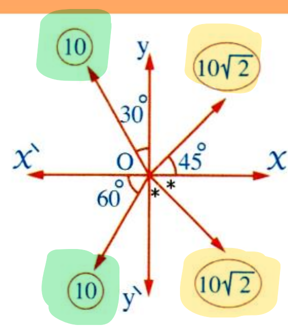
$\overrightarrow{R} = (4, 180^\circ)$

In the opposite figure :

The resultant of the forces (R) = …………… newton.

(a) 20 (b) $10\sqrt{2}$

(c) 10 (d) zero



$(10\sqrt{2}, 10\sqrt{2}) \Rightarrow$ [Two equal forces their resultant will act in East direction $= 2(10\sqrt{2})\cos 45$

$= 20$

$(10, 10) \Rightarrow$ [Two equal forces their resultant will act in West direction $= 2(10)\cos 60$

$= 10$

$\therefore R = 20 - 10 = 10$

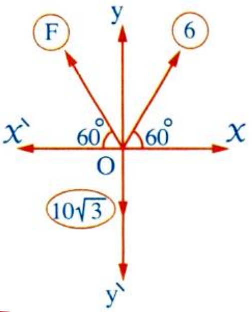
If the resultant of the forces represented in the opposite figure acts in X-axis , then F = ... newton.

(a) 10

(b) 14

(c) 18

(d) 6



$\because$ Resultant acts in X-axis $\Rightarrow \therefore \boxed{y=0}$

$(6, 60)$, $(F, 120^\circ)$, $(10\sqrt{3}, 270)$

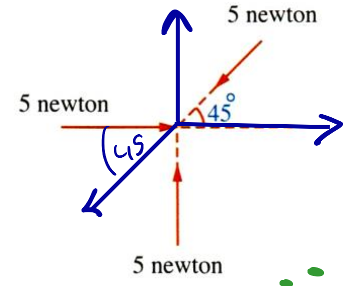
$$y = 6\sin 60 + F\sin 120 + 10\sqrt{3}\sin 270 = 0$$

$$\frac{\sqrt{3}}{2}F - 7\sqrt{3} = 0$$

$$\frac{\sqrt{3}}{2}F = 7\sqrt{3}$$

$$F = 14\ N$$

The opposite figure represents some of forces meeting at a point , then the magnitude of the resultant of these forces = ... newton.



(a) $15\sqrt{2}$

(b) 5

(c) $5\sqrt{2}-5$

(d) zero

خد بالك

من اتجاهات الأسهم

$(5, 0^\circ)$, $(5, 90^\circ)$, $(5, 225^\circ)$

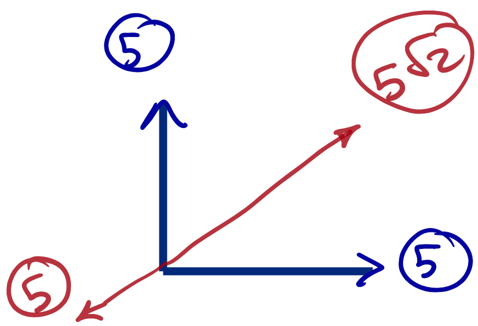
$x = 5\cos 0 + 5\cos 90 + 5\cos 225 = \dfrac{10-5\sqrt{2}}{2}$

$y = 5\sin 0 + 5\sin 90 + 5\sin 225 = \dfrac{10-5\sqrt{2}}{2}$

$R = \sqrt{x^2+y^2} = 5\sqrt{2}-5$

Another sol.

5 , 5 Two equal Perpendicular Forces their resultant $= 5\sqrt{2}$



$5\sqrt{2} > 5$

$\therefore R = 5\sqrt{2} - 5$

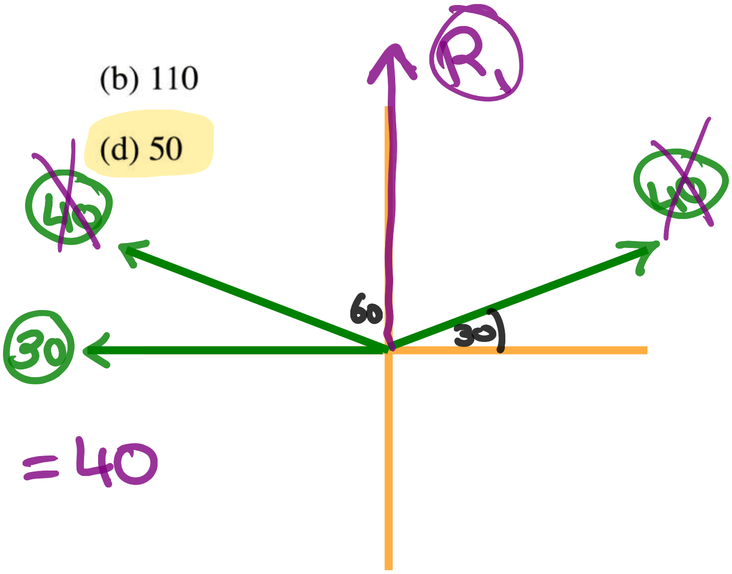
Three coplanar forces meeting at a point , their magnitudes are 40 , 30 , 40 newton , the first is in direction 60° West of North , the second is towards West and the third in the direction 30° North of East , then the magnitude of their resultant equal ·········· newton.

(a) 30

(b) 110

(c) 60

(d) 50



Resultant of (40,40)

$R_1 = 2F\cos\frac{\alpha}{2}$

$R_1 = 2(40)\cos 60 = 40$

$(R_1, 30)$ are Perpendicular

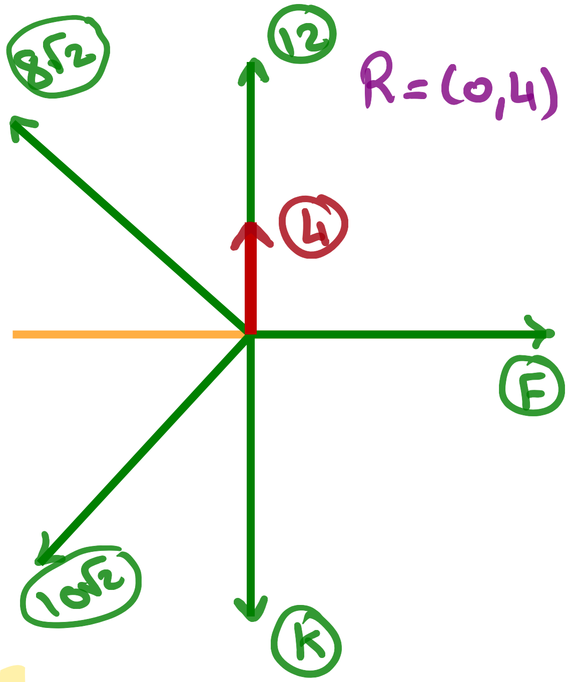
$R = \sqrt{(40)^2 + (30)^2} = 50$ N

The forces of magnitudes F , 12 , $8\sqrt{2}$, $10\sqrt{2}$, k newton act on a particle in the directions of East , North , Western North , Western South and South respectively. If the magnitude of the resultant = 4 newton due to North , then F – K = ... newton

(a) 24 (b) 27

(c) 12 (d) 6

$R = (0, 4)$



$(F, 0^\circ), (12, 90^\circ), (8\sqrt{2}, 135^\circ)$

$(10\sqrt{2}, 225^\circ), (K, 270^\circ)$

$X = F\cos 0^\circ + 12\cancel{\cos 90} + 8\sqrt{2}\cos 135^\circ + 10\sqrt{2}\cos 225^\circ + K\cancel{\cos 270} = 0$

$F - 18 = 0 \Rightarrow F = 18\ N$

$y = F\cancel{\sin 0^\circ} + 12\sin 90 + 8\sqrt{2}\sin 135^\circ + 10\sqrt{2}\sin 225 + K\sin 270 = 4$

$10 - K = 4 \Rightarrow K = 6\ N$

$F - K = 18 - 6 = 12\ N$

The coplanar forces of magnitudes 5 , 4 , F , 3 , k , 7 kg.wt. act at a particle and the measure of the angle between each two consecutive forces is 60° , if the system is in equilibrium (0,0) , then F + 2 K = 9 + 12 = 21 kg.wt.

(a) 21 (b) 6 (c) 9 (d) 15

(5, 0)

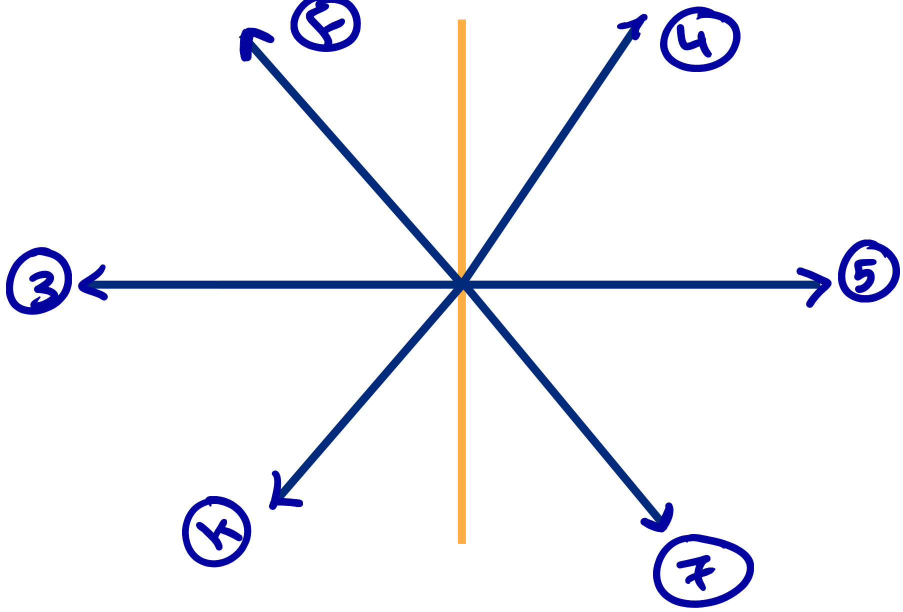
(4, 60)

(F, 120)

(3, 180)

(K, 240)

(7, 300)



$$X = 5\cos 0 + 4\cos 60 + F\cos 120 + 3\cos 180 + K\cos 240 + 7\cos 300 = 0$$

$$-\frac{1}{2}F - \frac{1}{2}K + \frac{15}{2} = 0 \qquad (\times -2)$$

$$\boxed{F + K = 15} \rightarrow \text{I}$$

$$y = 5\sin 0 + 4\sin 60 + F\sin 120 + 3\sin 180 + K\sin 240 + 7\sin 300 = 0$$

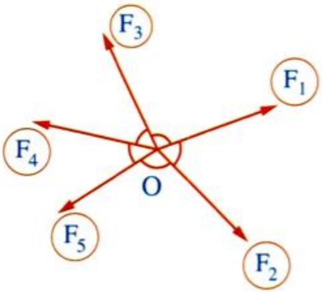
$$\frac{\sqrt{3}}{2}F - \frac{\sqrt{3}}{2}K - \frac{3\sqrt{3}}{2} = 0 \qquad \left(\div \frac{\sqrt{3}}{2}\right)$$

$$\boxed{F - K = 3} \rightarrow \text{II}$$

From I & II

F = 9   K = 6

The opposite figure represents a set of forces meeting at a point (O) Mohamed took (O) as an origin of coordinate system and the positive direction of X-axis in direction of $\overrightarrow{F_1}$ The magnitude of the resultant was R_1 and made angle of measure (θ_1) with the positive direction of X-axis and Ebrahim took (O) as an origin of coordinate system and the positive direction of X-axis in direction of $\overrightarrow{F_2}$, the magnitude of the resultant was R_2 and made an angle of measure (θ_2) with the positive direction of X-axis, then ....



(a) $R_1 = R_2$, $\theta_1 = \theta_2$

(b) $R_1 = R_2$, $\theta_1 \neq \theta_2$

(c) $R_1 \neq R_2$, $\theta_1 = \theta_2$

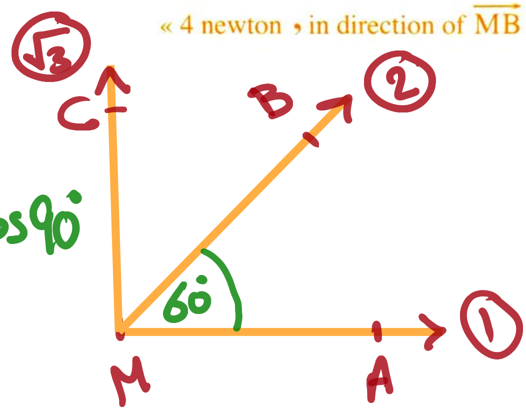
(d) $R_1 \neq R_2$, $\theta_1 \neq \theta_2$

Exercise 3

The resultant of coplanar forces meeting at a point (part 1)

Answer each of the following questions

(1) Three coplanar forces of magnitudes 1 , 2 , $\sqrt{3}$ newton act at M , their directions are $\overrightarrow{MA}$, $\overrightarrow{MB}$ and $\overrightarrow{MC}$ respectively where m ($\angle$ AMB) = 60° , m ($\angle$ BMC) = 30° , m ($\angle$ AMC) = 90° , find the resultant. « 4 newton , in direction of $\overrightarrow{MB}$ »



$(1, 0°), (2, 60°), (\sqrt{3}, 90°)$

$X = 1\cos 0° + 2\cos 60 + \sqrt{3}\cos 90°$
$= 2$

$y = 1\sin 0° + 2\sin 60 + \sqrt{3}\sin 90$
$= 2\sqrt{3}$

$\overrightarrow{R} = (2, 2\sqrt{3}) \in 1^{st}$ quad.

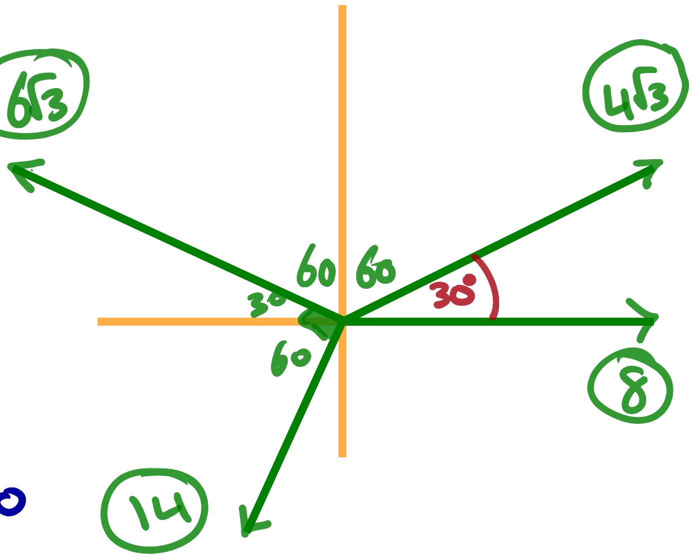
$\|\overrightarrow{R}\| = \sqrt{(2)^2 + (2\sqrt{3})^2} = 4\ N$

$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = 60°$ $\quad \overrightarrow{R} = (4N, 60°)$

② The forces 8 , $4\sqrt{3}$, $6\sqrt{3}$ and 14 newton act at a point , the measure of the angle between the first force and the second force is 30° , between the second and the third is 120° and between the third and the fourth is 90° taken in the same cyclic order. Find the magnitude and direction of the resultant of these forces.

« 4 newton , in direction of 4[th] force »

$(8,0), (4\sqrt{3}, 30), (6\sqrt{3}, 150^\circ)$
$(14, 240^\circ)$



$x = 8\cos 0^\circ + 4\sqrt{3}\cos 30$
$+ 6\sqrt{3}\cos 150 +$
$14\cos 240 = -2$

$y = 8\sin 0^\circ + 4\sqrt{3}\sin 30$
$+ 6\sqrt{3}\sin 150 + 14\sin 240 = -2\sqrt{3}$

$\vec{R} = -2\hat{i} - 2\sqrt{3}\hat{j} \in 3^{rd}$ quad.

$\|\vec{R}\| = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = 4$ N

$\theta = 180^\circ + \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = 240^\circ$

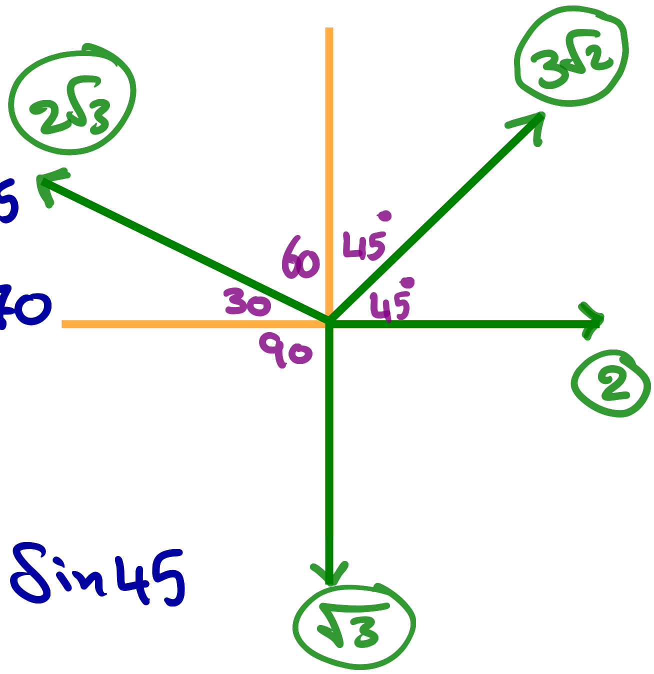
$\vec{R} = (4N, 240^\circ)$

(3) The coplanar forces of magnitudes 2 , $3\sqrt{2}$, $2\sqrt{3}$ and $\sqrt{3}$ newton act at a point. If the measures between the first force and the second force is 45° , the measure between the second and the third is 105° and the measure between the third and the fourth is 120° taken in the same cyclic order , find the resultant of these forces.

«$\sqrt{13}$ newton , 11° 19̀ with 2nd force»

$(2, 0^\circ)$, $(3\sqrt{2}, 45)$

$(2\sqrt{3}, 150^\circ)$, $(\sqrt{3}, 270^\circ)$



$X = 2\cos 0 + 3\sqrt{2}\cos 45 + 2\sqrt{3}\cos 150 + \sqrt{3}\cos 270$

$= 2$

$y = 2\sin 0 + 3\sqrt{2}\sin 45 + 2\sqrt{3}\sin 150^\circ + \sqrt{3}\sin 270$

$= 3$

$\vec{R} = 2\hat{i} + 3\hat{j}$

$\|\vec{R}\| = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$ N

$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56^\circ\ 18'\ 36''$

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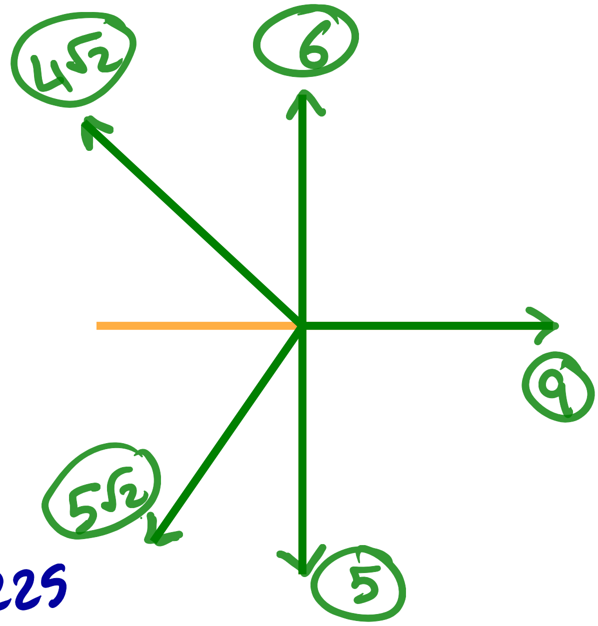
4 Five coplanar forces meeting at a point , their magnitudes are 9 , 6 , $4\sqrt{2}$, $5\sqrt{2}$ and 5 newton act due to East , North , Western North , Western South and in the direction of South respectively. Prove that the set of forces are in equilibrium.

$(9, 0°)$, $(6, 90°)$, $(4\sqrt{2}, 135°)$, $(5\sqrt{2}, 225°)$, $(5, 270°)$



$X = 9\cos 0 + 6\cos 90 + 4\sqrt{2}\cos 135 + 5\sqrt{2}\cos 225 + 5\cos 270 = 0$

$y = 9\sin 0 + 6\sin 90 + 4\sqrt{2}\sin 135° + 5\sqrt{2}\sin 225 + 5\sin 270 = 0$

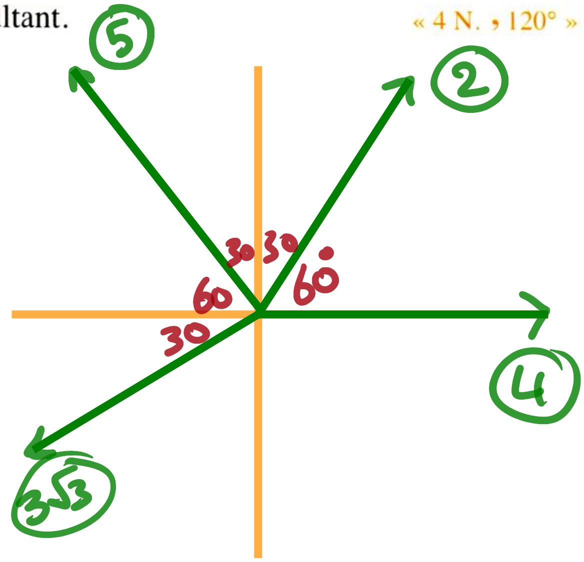
$\vec{R} = \vec{0}$

$\therefore$ The set of forces are equilibrium.

(5) Four coplanar forces act on a particle the first of magnitude 4 newton acts in the Eastern direction , the second of magnitude 2 newton , acts in direction 60° North of the East , the third of magnitude 5 newton , acts in direction 60° North of the West and the fourth of magnitude $3\sqrt{3}$ newton acts in direction 60° West of the South.
Find the magnitude and direction of their resultant. « 4 N. , 120° »

$(4, 0^\circ), (2, 60^\circ)$

$(5, 120^\circ), (3\sqrt{3}, 210^\circ)$



$X = 4\cos 0^\circ + 2\cos 60 + 5\cos 120 + 3\sqrt{3}\cos 210 = -2$

$y = 4\sin 0^\circ + 2\sin 60^\circ + 5\sin 120 + 3\sqrt{3}\sin 210^\circ = 2\sqrt{3}$

$\vec{R} = -2\hat{i} + 2\sqrt{3}\hat{j}$ $\in 2^{nd}$ quad.

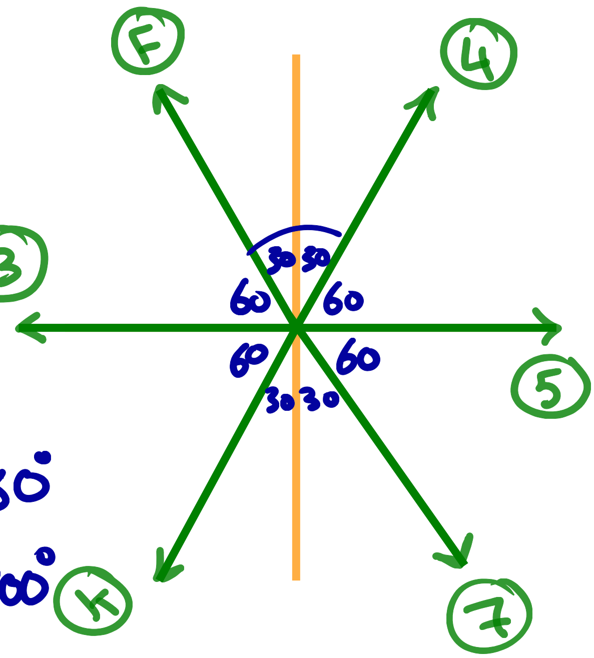
$\|\vec{R}\| = \sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$ N

$\theta = 180^\circ - \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = 120^\circ$

$\vec{R} = (4\text{ N}, 120^\circ)$

⑥ The coplanar forces of magnitudes 5 , 4 , F , 3 , K and 7 kg.wt. act at a particle and the measure of the angle between each two consecutive forces is 60°
Find the magnitude of F and K that makes the system in equilibrium. « 9 , 6 kg.wt. »

$(5, 0^\circ), (4, 60^\circ)$
$(F, 120^\circ), (3, 180^\circ)$
$(K, 240^\circ), (7, 300^\circ)$



$$X = 5\cos 0^\circ + 4\cos 60 + F\cos 120^\circ + 3\cos 180^\circ + K\cos 240^\circ + 7\cos 300^\circ = \text{zero.}$$

$$-\frac{1}{2}F - \frac{1}{2}K + \frac{15}{2} = 0 \qquad \boxed{\times -2}$$

$$F + K = 15 \rightarrow (1)$$

$$y = 5\sin 0^\circ + 4\sin 60 + F\sin 120^\circ + 3\sin 180^\circ + K\sin 240 + 7\sin 300 = 0$$

$$\frac{\sqrt{3}}{2}F - \frac{\sqrt{3}}{2}K - \frac{3\sqrt{3}}{2} = 0 \qquad \boxed{\div \frac{\sqrt{3}}{2}}$$

$$F - K = 3 \rightarrow (2)$$

$F = 9$ kg.wt. $K = 6$ kg.wt

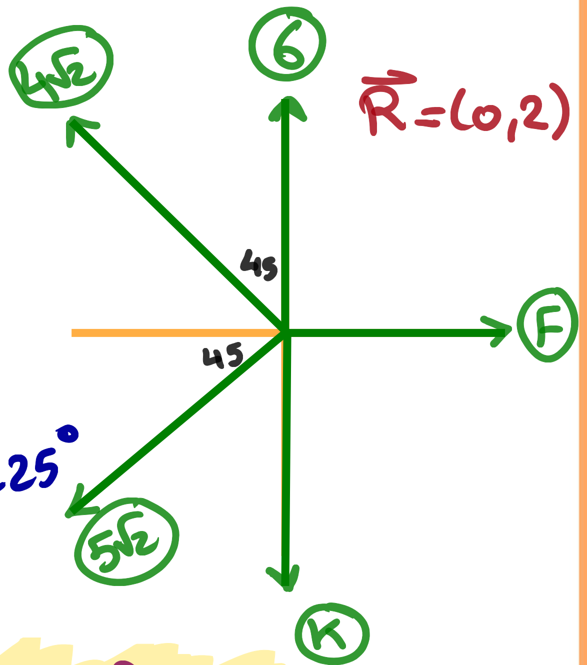
(7) The forces of magnitudes F , 6 , $4\sqrt{2}$, $5\sqrt{2}$, K newton act on a particle in the directions of East , North , Western North , Western South and South respectively. Find the values of F and K if the magnitude of the resultant = 2 newton due to North.

« 9 , 3 newton »

$(F, 0^\circ), (6, 90^\circ)$

$(4\sqrt{2}, 135^\circ), (5\sqrt{2}, 225^\circ)$

$(K, 270^\circ)$



$x = F\cos 0^\circ + 6\cos 90^\circ + 4\sqrt{2}\cos 135^\circ + 5\sqrt{2}\cos 225^\circ + K\cos 270^\circ = 0$

$F - 9 = 0 \Rightarrow F = 9$ Newton

$y = F\sin 0^\circ + 6\sin 90^\circ + 4\sqrt{2}\sin 135^\circ + 5\sqrt{2}\sin 225^\circ + K\sin 270 = 2$

$5 - K = 2 \Rightarrow K = 3$ Newton

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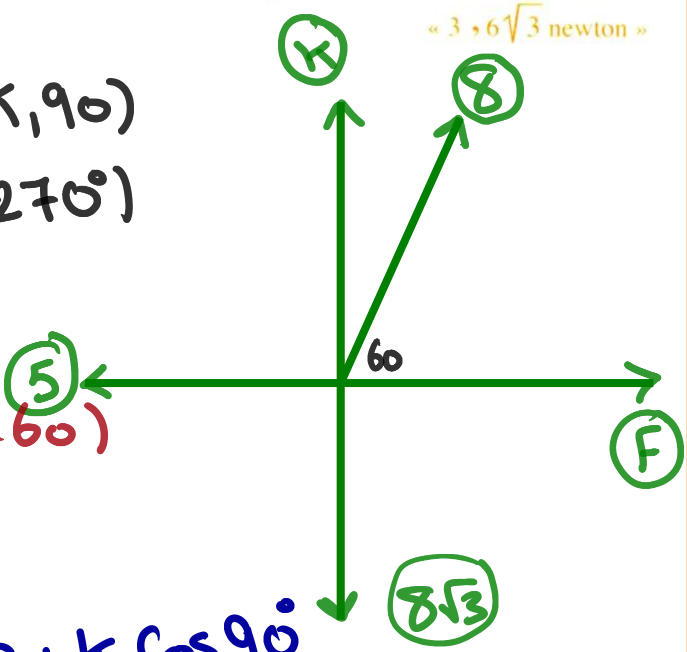
8. The forces of magnitudes F , 8 , K , 5 , $8\sqrt{3}$ newton act at a point in the directions of : East , 30° East of North , North , West and South respectively.
Find the values of F and K if the resultant is 4 newton in magnitude in the direction of 60° North of East. « 3 , $6\sqrt{3}$ newton »

$(F, 0^\circ), (8, 60^\circ), (K, 90)$
$(5, 180^\circ), (8\sqrt{3}, 270^\circ)$

$\vec{R} = (4, 60^\circ)$
$= (4\cos 60, 4\sin 60)$
$= (2, 2\sqrt{3})$



$X = F\cos 0^\circ + 8\cos 60 + K\cos 90^\circ$
$+ 5\cos 180 + 8\sqrt{3}\cos 270 = 2$

$F - 1 = 2 \Rightarrow F = 3\text{ N}$

$y = F\sin 0^\circ + 8\sin 60^\circ + K\sin 90$
$+ 5\sin 180 + 8\sqrt{3}\sin 270 = 2\sqrt{3}$

$K - 4\sqrt{3} = 2\sqrt{3} \Rightarrow K = 6\sqrt{3}\text{ N}$