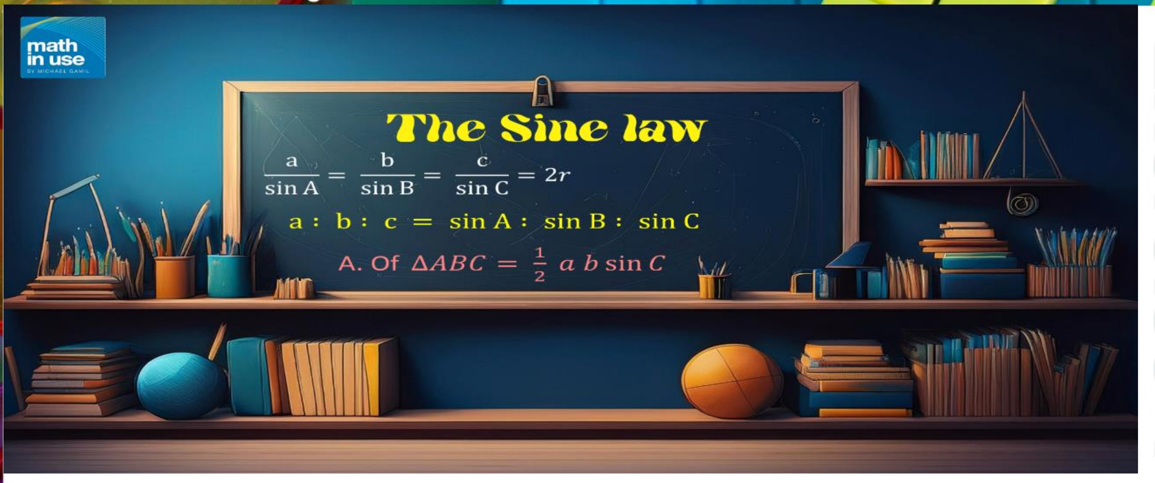


Exercise 1

The Sine rule

Choose the correct answer

In any triangle XYZ , XY : YZ = ……………

(a) sin X : sin Y (b) sin Y : sin Z (c) sin Z : sin X (d) sin Z : sin Y

XY : YZ = z : x

= Sin Z : Sin X

In Δ ABC , if m (∠ A) = 30° , C = $15\sqrt{3}$ cm. , m (∠ C) = 60° , then a = ... cm.

(a) 30 (b) 45 (c) 15 (d) 60

$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{a}{\sin 30} = \frac{15\sqrt{3}}{\sin 60}$$

$$a = \frac{15\sqrt{3}\,\sin 30}{\sin 60} = 15 \text{ cm}$$

DEF is a triangle in which m (∠ D) = 80° and m (∠ E) = 60° , if f = 12 cm.,
then d = ... cm.

(a) $\frac{12 \sin 80°}{\sin 40°}$ (b) $\frac{12 \sin 80°}{\sin 60°}$ (c) $\frac{12 \sin 40°}{\sin 80°}$ (d) $\frac{12 \cos 80°}{\cos 40°}$

$m(\angle F) = 180 - (80 + 60) = 40$

$\frac{d}{\sin D} = \frac{f}{\sin F} \Rightarrow \frac{d}{\sin 80} = \frac{12}{\sin 40}$

$d = \frac{12 \sin 80}{\sin 40}$

In Δ ABC , if a = 4 cm. , b = 7 cm. , m ($\angle$ C) = 120° , then the area of the triangle = ... cm^2.

(a) $7\sqrt{3}$ (b) $14\sqrt{3}$ (c) 7 (d) 14

A. of $\Delta ABC = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(4)(7) \sin 120$

$= 7\sqrt{3}$ cm^2

XYZ is an equilateral triangle , the length of its side is $10\sqrt{3}$ cm. , then the length of the diameter of its circumcircle is .... cm.

(a) 5 (b) 10 (c) 15 (d) 20

$$\frac{a}{\sin A} = 2r = \text{diameter}$$

$$\therefore \text{Diameter} = \frac{10\sqrt{3}}{\sin 60} = 20 \text{ cm}$$

In Δ XYZ , $\frac{x}{\sin X} = 6$, then the length of the diameter of its circumcircle is .... length units.

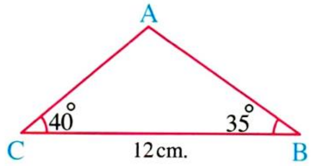
(a) 6 (b) 12 (c) 3 (d) 9

$\frac{x}{\sin X} = 2r = 6$

Diameter $= 6$

In the opposite figure :

The length of $\overline{AB} \approx$ .... cm.



(a) 6 (b) 7

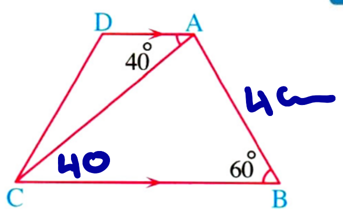
(c) 8 (d) 9

$m(\angle A) = 180 - (40 + 35) = 105°$

$\frac{AB}{\sin C} = \frac{BC}{\sin A} \Rightarrow \frac{AB}{\sin 40} = \frac{12}{\sin 105}$

$AB = \frac{12 \sin 40}{\sin 105} \simeq 8$ cm

In the opposite figure :



$\overline{AD} \parallel \overline{BC}$, AB = 4 cm. , m (∠ DAC) = 40° , m (∠ B) = 60°

, then the length of $\overline{AC} \simeq$ .... cm.

(a) 5 (b) 3

(c) 2 (d) 4

In $\triangle ABC$

$$\frac{AB}{\sin C} = \frac{AC}{\sin B} \Rightarrow \frac{4}{\sin 40} = \frac{AC}{\sin 60}$$

$$AC = \frac{4 \sin 60}{\sin 40} \simeq 5 \text{ cm}$$

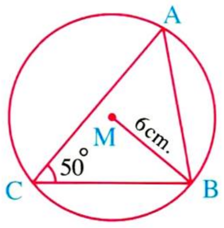
In the opposite figure :

M is the centre of the circle

BM = 6 cm. , then AB = ... cm.

(a) 6 sin 50° (b) 12 sin 50°

(c) 6 cos 50° (d) 12 cos 50°



$$\frac{AB}{\sin C} = 2r$$

$$\frac{AB}{\sin 50} = 2(6)$$

$$AB = 12 \sin 50°$$

A circle with diameter of length 20 cm. , passes through the vertices of Δ ABC which is an acute-angled triangle in which BC = 10 cm. , then m ($\angle$ A) = ...°

(a) 30 (b) 60 (c) 45 (d) 150

$a = BC = 10$ cm

$$\frac{a}{\sin A} = 2r$$

$$\frac{10}{\sin A} = \frac{20}{1} \Rightarrow \sin A = \frac{1}{2}$$

$$\therefore m(\angle A) = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

In triangle ABC , m ($\angle$ A) = 45° , the length of the radius of its circumcircle = 6 cm.
, then a = ·············· cm.

(a) 13 (b) $6\sqrt{2}$ (c) 12 (d) $\sqrt{2}$

$$\frac{a}{\sin A} = 2r \Rightarrow \frac{a}{\sin 45} = 2(6)$$

$$a = 12 \sin 45 = 6\sqrt{2} \text{ cm}$$

If the length of a side in any triangle = 12 cm. and the measure of the opposite angle to this side = 55° , then the circumference of the circle that passes through the vertices of this triangle ≃ .... cm.

(a) 36 (b) 42 (c) 46 (d) 52

$$\frac{a}{\sin A} = 2r \Rightarrow \therefore 2r = \frac{12}{\sin 55} \simeq 14.65 \text{ cm}$$

Circ. of the Circle = $2\pi r$

$= 14.65\,\pi \simeq 46$ cm

If the perimeter of triangle ABC equals 15 cm. , m (∠ A) = 53° , m (∠ B) = 47° , then the length of $\overline{AB} \simeq$ .... cm.

(a) 6 (b) 7 (c) 5 (d) 8

$m(\angle C) = 180 - [53 + 47] = 80°$

$$\frac{AB}{\sin C} = \frac{P.\text{ of } \triangle ABC}{\sin A + \sin B + \sin C}$$

$$\frac{AB}{\sin 80} = \frac{15}{\sin 53° + \sin 47° + \sin 80}$$

$$AB = \frac{15 \sin 80}{\sin 53 + \sin 47 + \sin 80} \simeq 6$$

In triangle ABC , a = 27 cm. , m (∠ B) = 82° , m (∠ C) = 56°

, then its surface area ≃ .... cm^2.

(a) 540 (b) 447 (c) 350 (d) 400

$m(\angle A) = 180 - [82 + 56]$

$= 42°$

$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{27}{\sin 42} = \frac{b}{\sin 82}$

$b = \frac{27 \sin 82}{\sin 42} \simeq 39.96$ cm

A. of $\triangle ABC = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(27)(39.96) \sin 56$

$\simeq 447$ cm^2

In triangle ABC , m (∠ A) : m (∠ B) : m (∠ C) = 2 : 3 : 4 , AB = 12 cm. , then the length of $\overline{AC}$ ≃ .... cm.

c

b

(a) 10 (b) 11 (c) 16 (d) 18

A : B : C : Sum

2 : 3 : 4 : 9

: : : 180

$m(\angle A) = 40^\circ$, $m(\angle B) = 60^\circ$, $m(\angle C) = 80^\circ$

$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{AC}{\sin 60} = \frac{12}{\sin 80}$$

$$AC = \frac{12 \sin 60}{\sin 80} = 11 \text{ cm}$$

In triangle ABC , which of the following statements is true ?

(a) $\sin A + \cos B = a + b$

(b) $a \sin B = b \sin A$

(c) $a = b \sin c$

(d) $\frac{a}{\sin A} = \frac{\sin B}{b}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow a \sin B = b \sin A$$

In Δ XYZ , 2 r sin X = ... "where r is the radius length of its circumcircle"

(a) z (b) y (c) x (d) area of Δ XYZ

$$\frac{x}{\sin X} = \frac{2r}{1} \Rightarrow x = 2r\sin X$$

If r is the length of the radius of the circumcircle of the triangle XYZ ,

then $\frac{y}{2 \sin Y}$ = ……………

(a) r (b) 2 r (c) $\frac{1}{2}$ r (d) 4 r

$$\frac{y}{\sin Y} = 2r \Longrightarrow \therefore r = \frac{y}{2 \sin Y}$$

In acute-angled triangle ABC , $2\,a = \frac{b}{\sin B}$, then m (∠ A) = ……………

(a) 30° (b) 45° (c) 60° (d) 75°

$\because \frac{a}{\sin A} = \frac{b}{\sin B}$ & $\because 2a = \frac{b}{\sin B}$ "given"

$\therefore \frac{2a}{1} = \frac{a}{\sin A}$

$\sin A = \frac{a}{2a} = \frac{1}{2}$

$m(\angle A) = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

In Δ ABC , sin A = 2 sin C , BC = 6 cm. , then AB = ... cm.

(a) 2 (b) 3 (c) 4 (d) 6

$1 \sin A = 2 \sin C$

$$\frac{\sin A}{\sin C} = \frac{2}{1} \Rightarrow \therefore \frac{a}{c} = \frac{2}{1}$$

$$\therefore \frac{BC}{AB} = \frac{2}{1}$$

$$\therefore \frac{6}{AB} = \frac{2}{1} \Rightarrow \therefore AB = 3\text{cm}$$

If the radius length of circumcircle of Δ ABC equals 3 cm.
and sin A + sin B + sin C = 2, then the perimeter of triangle ABC = ... cm.

(a) 6 (b) 9 (c) 12 (d) 24

$$\frac{\text{P. of } \Delta ABC}{\sin A + \sin B + \sin C} = 2r$$

$$\text{P. of } \Delta ABC = 2r\,[\sin A + \sin B + \sin C]$$

$$= 2(3)\,[2] = 12 \text{ cm}$$

ABC is an equilateral triangle , its side length is 6 cm. and the area of its circumcircle equals k π cm^2 , then k = ...

(a) $2\sqrt{3}$ (b) $8\sqrt{3}$ (c) 12 (d) 24

$$\frac{x}{\sin X} = 2r \Rightarrow r = \frac{6}{2 \sin 60}$$

$$\therefore r = 2\sqrt{3}$$

A. of the circle $= \pi r^2 = \pi (2\sqrt{3})^2$

$= 12 \pi \text{ cm}^2$

$= k \pi \text{ cm}^2$

$\boxed{\therefore k = 12}$

In any triangle ABC , $\frac{\sin(A+B)}{\sin A + \sin B} = \dots\dots\dots\dots$

(a) 1 (b) $\frac{c}{a+b}$ (c) $\frac{a}{b+c}$ (d) $\frac{b}{a+c}$

$$\frac{\sin(A+B)}{\sin A+\sin B} = \frac{\sin C}{\sin A+\sin B}$$

$$= \frac{c}{a+b}$$

[Ratio bet. Sine of angle = Ratio bet. Sides]

Note that

$A+B+C=180$

$A+B=180-C$

$\sin(A+B)=\sin(180-C)$

$\sin(A+B)=\sin C$

In Δ ABC , $\frac{a}{a+b} = \frac{\sin A}{\cdots\cdots\cdots}$

(a) sin B (b) sin C (c) sin A + sin B (d) sin A + sin C

$$\frac{a}{a+b} = \frac{\sin A}{\sin A + \sin B}$$

In ΔXYZ, if $3 \sin X = 4 \sin Y = 2 \sin Z$, then $x : y : z =$ ……………

(a) 2 : 3 : 4 (b) 6 : 4 : 3 (c) 3 : 4 : 6 (d) 4 : 3 : 6

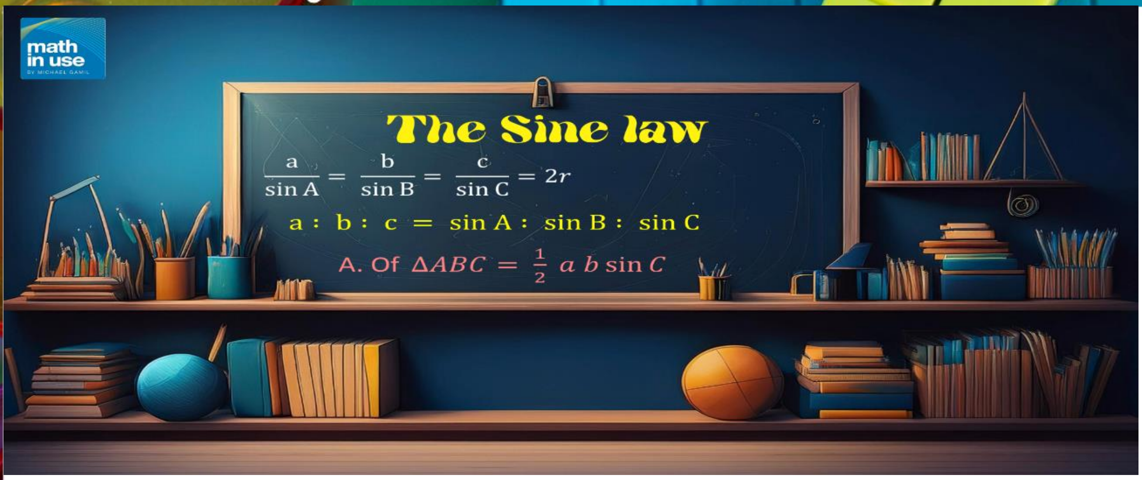
$$\frac{3 \sin X}{12} = \frac{4 \sin Y}{12} = \frac{2 \sin Z}{12}$$

$$\frac{\sin X}{4} = \frac{\sin Y}{3} = \frac{\sin Z}{6}$$

$$\therefore x : y : z = 4 : 3 : 6$$

2, 3, 4	2
1, 3, 2	2
1, 3, 1	3
1, 1, 1	

L.C.M = 12



Exercise 1

The Sine rule

ABC is a triangle in which $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$, then a : b : c = ...

(a) 6 : 5 : 8 (b) 8 : 5 : 6 (c) 7 : 2 : 4 (d) 3 : 5 : 4

$$\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4} \qquad (\div 2)$$

$$\frac{\sin A}{6} = \frac{\sin B}{5} = \frac{\sin C}{8}$$

In ΔABC : If $\frac{\sin A}{4} = \frac{\sin B}{9} = \frac{\sin C}{7}$, then the greatest angle in measure is ....

(a) $\angle A$ (b) $\angle B$ (c) $\angle C$ (d) right

$$\frac{\sin A}{4} = \frac{\sin B}{9} = \frac{\sin C}{7}$$

$$\therefore a : b : c = 4 : 9 : 7$$

greatest angle Opposite to greatest side

In triangle ABC , m (∠ A) : m (∠ B) : m (∠ C) = 3 : 5 : 4 , then $c^2 : a^2 =$ ....

(a) $\sqrt{6} : 2$ (b) $2 : 3$ (c) $4 : 3$ (d) $3 : 2$

A : B : C : Sum

3 : 5 : 4 : 12

? : ? : ? : 180

$m(\angle A) = 45^\circ$ $m(\angle B) = 75^\circ$ $m(\angle C) = 60^\circ$

$c : a = \sin C : \sin A$

$= \sin 60 : \sin 45$

$= \sqrt{6} : 2$

$\therefore c^2 : a^2 = 6 : 4 = \boxed{3 : 2}$

In Δ ABC , $\frac{a}{b} \times \frac{\sin B}{\sin A} =$ ....

(a) $\frac{c}{\sin C}$ (b) $\frac{\sin C}{c}$ (c) $4r$ (d) 1

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{a}{b} = \frac{\sin A}{\sin B}$$

$$\therefore \frac{a}{b} \times \frac{\sin B}{\sin A}$$

$$= \frac{\sin A}{\sin B} \times \frac{\sin B}{\sin A} = 1$$

In ΔABC , if the radius of its circumcircle = 4 cm.

, then $\dfrac{a+b+c}{\sin A + \sin B + \sin C} =$ .... $2r = 2(4) = 8$

(a) 4 (b) 2 (c) 8 (d) 16

If Δ ABC is a right-angled at $\angle$ B and b = 10 cm.

, then $\frac{a}{\sin A} + \frac{c}{\sin C} = \dots\dots\dots\dots$ cm.

(a) 10 (b) 20 (c) 40 (d) 100

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\therefore \frac{a}{\sin A} + \frac{c}{\sin C} = \frac{b}{\sin B} + \frac{b}{\sin B}$$

$$= \frac{2b}{\sin B} = \frac{2(10)}{\sin 90} = 20$$

math in use BY MICHAEL GAMIL

If the radius of the circumcircle of Δ ABC equals r , then the perimeter of the triangle = ... (sin A + sin B + sin C)

(a) r (b) 2 r (c) 4 r^2 (d) 8 r^3

$$\frac{\text{P. of } \Delta ABC}{\sin A + \sin B + \sin C} = \frac{2r}{1}$$

$$\text{P. of } \Delta ABC = 2r\,[\sin A + \sin B + \sin C]$$

In Δ ABC , a – b = 4 cm. , $\sin A = \frac{3}{2} \sin B$, then a = ... cm.

(a) 4 (b) 6 (c) 8 (d) 12

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\frac{3}{2}\cancel{\sin B}} = \frac{b}{\cancel{\sin B}} \Rightarrow \therefore \frac{a}{3/2} = \frac{b}{1}$$

$$a = \frac{3}{2} b \Rightarrow 2a = 3b$$

$$2a - 3b = 0$$

$$a - b = 4 \quad \text{given}$$

$$a = 12 \quad , \quad b = 8$$

If the perimeter of Δ ABC is 24 cm. and sin A + sin B = 3 sin C , then C = ·············· cm.

(a) 4 (b) 6 (c) 8 (d) 9

$$\frac{C}{\sin C} = \frac{\text{P. of } \Delta ABC}{\sin A + \sin B + \sin C}$$

$$\frac{C}{\sin C} = \frac{24}{3\sin C + \sin C}$$

$$\frac{C}{\cancel{\sin C}} = \frac{24}{4\cancel{\sin C}}$$

$$C = 6 \text{ cm}$$

ABC is a triangle , sin B + sin C = 4 sin A and b + c = 2 a + 10 cm.
, then a = ... cm.

(a) 2 (b) 3 (c) 4 (d) 5

$$\frac{a}{\sin A} = \frac{a + b + c}{\sin A + \sin B + \sin C}$$

$$\frac{a}{\sin A} = \frac{a + 2a + 10}{\sin A + 4\sin A}$$

$$\frac{a}{\cancel{\sin A}} = \frac{3a + 10}{5\cancel{\sin A}}$$

$$5a = 3a + 10$$

$$2a = 10$$

$$\therefore a = 5$$

In Δ ABC , AB = 8 cm. , BC = 12 cm. , m ($\angle$ A) – m ($\angle$ C) = 90°

, then tan C = ...

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

$AB = c$, $BC = a$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{12}{\cos C} = \frac{8}{\sin C}$$

$$\frac{\sin C}{\cos C} = \frac{8}{12}$$

$$\tan C = \frac{2}{3}$$

$$A - C = 90$$

$$A = 90 + C$$

$$\sin A = \sin(90 + C)$$

$$\sin A = \cos C$$

If r is the radius length of the circumcircle of Δ ABC and a = r

, then m (∠ A) = ...

(a) 30° only. (b) 30° or 120° (c) 150° only. (d) 30° or 150°

$$\frac{a}{\sin A} = \frac{2r}{1} \qquad \boxed{a = r}$$

$$\frac{r}{\sin A} = \frac{2r}{1}$$

$$\sin A = \frac{\cancel{r}}{2\cancel{r}} = \frac{1}{2}$$

1st quad. θ

2nd quad. $180 - \theta$

$$m(\angle A) = \sin^{-1}\frac{1}{2}$$

$$m(\angle A) = 30^\circ \quad \text{or} \quad m(\angle A) = 180 - 30 = 150^\circ$$

If the area of the triangle ABC is Δ and r is the radius length of the circumcircle of the triangle ABC , then : $\frac{4\,r\,\Delta}{abc}$ = ……………

(a) 1 (b) 2 (c) 4 (d) 8

$A.$ of $\Delta ABC = \frac{1}{2} ab \sin C$

$$\therefore \frac{4r\Delta}{abc} = \frac{4r(\frac{1}{2}\cancel{a}\cancel{b}\sin C)}{\cancel{a}\cancel{b}c}$$

$$= \frac{2r\sin C}{c}$$

" but $\frac{c}{\sin C} = 2r$ "

$\therefore \frac{\sin C}{c} = \frac{1}{2r}$

$$= 2r \cdot \frac{1}{2r} = 1$$

In Δ ABC , $\frac{2\,b}{\sin B}$ = ... r (where r is the radius of its circumcircle)

(a) 1 (b) 2 (c) 4 (d) 8

$\because \frac{b}{\sin B} = 2r$

$\therefore \frac{2b}{\sin B} = 2(2r) = 4r$

ABC is a triangle , b = 12 cm. , the radius length of its circumcircle is r , then the area of the triangle = ... cm^2.

(a) $\frac{2\,ac}{r}$ (b) $\frac{3\,ac}{r}$ (c) $\frac{4\,ac}{r}$ (d) $\frac{6\,ac}{r}$

$$\frac{b}{\sin B} = \frac{2r}{1} \Rightarrow \sin B = \frac{b}{2r} = \frac{12}{2r}$$

$$\boxed{\therefore \sin B = \frac{6}{r}}$$

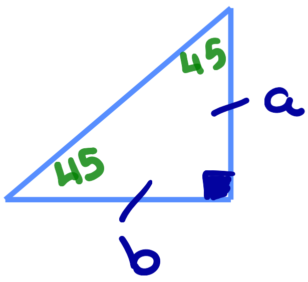
$$A \text{ of } \triangle ABC = \frac{1}{2} ac \sin B$$

$$= \frac{1}{2} ac \cdot \frac{6}{r} = \frac{3ac}{r}$$

If the triangle ABC is an isosceles right-angled triangle and r is the radius length of the circumcircle of the triangle ABC , then the area of Δ ABC = ... (in terms of r)

(a) $\frac{1}{2}$ r^2 (b) 2 r^2 (c) r^2 (d) 4 r^2

$$r = \frac{a}{2\sin 45} = \frac{b}{2\sin 45} = \frac{c}{2\sin 90}$$



$$r = \frac{a}{\sqrt{2}} = \frac{b}{\sqrt{2}} = \frac{c}{2}$$

$$\therefore a = b = \sqrt{2}\,r$$

$$\text{A. of } \Delta ABC = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(\sqrt{2}r)(\sqrt{2}r)\sin 90$$

$$= r^2$$

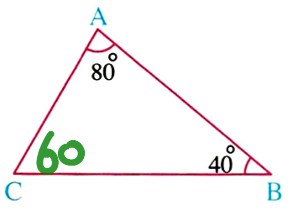
In the opposite figure :

If the perimeter of Δ ABC = 20 cm. ,

then the diameter length of its circumcircle ≈ .... cm.

(a) 2 (b) 4

(c) 6 (d) 8



$$\frac{\text{P. of } \Delta ABC}{\sin A + \sin B + \sin C} = 2r$$

$$2r = \frac{20}{\sin 80 + \sin 40 + \sin 60}$$

Diameter ≃ 8 cm

In Δ ABC , cos (B + C) = $\frac{3}{5}$, BC = 8 cm. , then the radius length of the circumcircle of Δ ABC = ... cm.

(a) 4 (b) 5 (c) 8 (d) 10

$$\frac{a}{\sin A} = 2r$$

$$r = \frac{a}{2\sin A}$$

$$r = \frac{8}{2\left(\frac{4}{5}\right)} = 5$$

$$A + B + C = 180$$

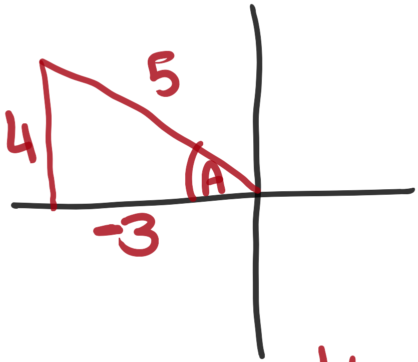
$$B + C = 180 - A$$

$$\cos(B+C) = \cos(180 - A)$$

$$\cos(B+C) = -\cos A$$

$$\frac{3}{5} = -\cos A$$

$$\cos A = -\frac{3}{5}$$



$$\sin A = \frac{4}{5}$$

If the area of a triangle is $\dfrac{a^2 \sin B \sin C}{k \sin A}$, then $k =$ .... , $k \neq 0$

(a) 1 (b) 2 (c) 3 (d) 4

$A.$ of $\triangle ABC = \frac{1}{2} ab \sin C$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\boxed{b = \frac{a \sin B}{\sin A}}$

$A = \frac{1}{2} a \left(\frac{a \sin B}{\sin A}\right) \sin C$

$= \dfrac{a^2 \sin B \sin C}{2 \sin A}$

$\boxed{\therefore K = 2}$

math in use
BY MICHAEL GAMIL

If r is the radius length of the circumcircle of triangle ABC and $a = \frac{1}{2}$ r , then m (∠ A) = ... where A is an acute angle.

(a) $\sin^{-1}(1)$ (b) $\sin^{-1}(2)$ (c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{1}{4}\right)$

$$\frac{a}{\sin A} = 2r \Rightarrow \frac{\frac{1}{2}\cancel{r}}{\sin A} = \frac{2\cancel{r}}{1}$$

$$\sin A = \frac{1 \times \frac{1}{2}}{2} = \frac{1}{4}$$

$$m(\angle A) = \sin^{-1}\left(\frac{1}{4}\right)$$

If Δ is the area of triangle XYZ , S is half the perimeter of the triangle XYZ , then $\frac{2}{z \sin X} + \frac{2}{x \sin Y} + \frac{2}{y \sin Z} = \ldots\ldots\ldots\ldots$

(a) $\frac{S}{\Delta}$ (b) $\frac{2S}{\Delta}$ (c) $\frac{3S}{\Delta}$ (d) $\frac{4S}{\Delta}$

$$\frac{2(\frac{1}{2}y)}{\frac{1}{2}yz\sin x} + \frac{2(\frac{1}{2}z)}{\frac{1}{2}zx\sin y} + \frac{2(\frac{1}{2}x)}{\frac{1}{2}xy\sin z}$$

$$\frac{y}{\Delta} + \frac{z}{\Delta} + \frac{x}{\Delta} = \frac{x+y+z}{\Delta}$$

$$= \frac{\text{Per.}}{\Delta} = \frac{2S}{\Delta}$$

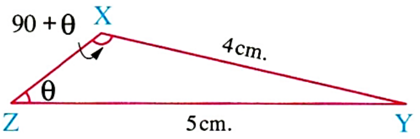
In the opposite figure $\tan\theta =$ ....

(a) $\frac{3}{5}$

(b) $\frac{4}{3}$

(c) $\frac{5}{4}$

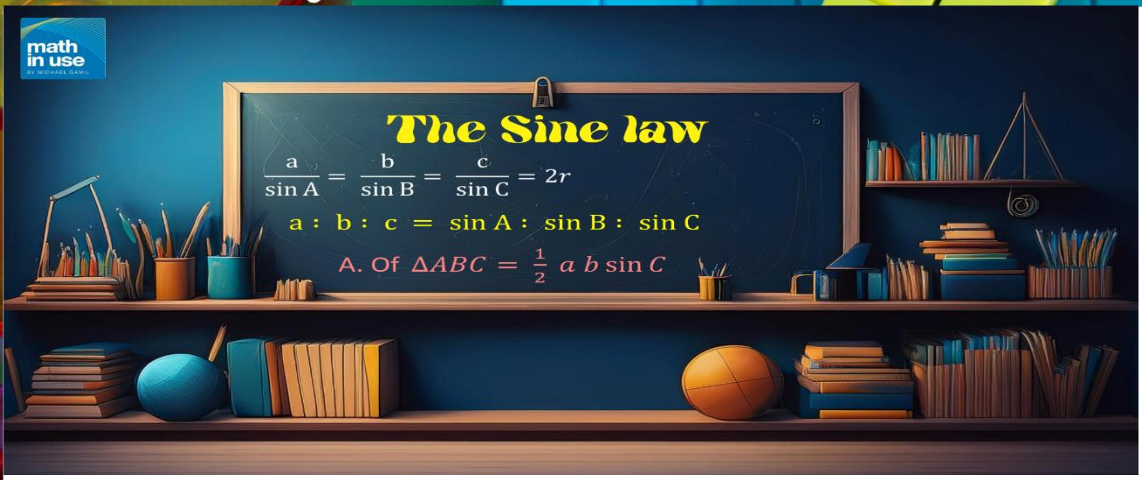
(d) $\frac{4}{5}$



$$\frac{x}{\sin X} = \frac{z}{\sin Z} \Rightarrow \frac{5}{\sin(90+\theta)} = \frac{4}{\sin\theta}$$

$$\therefore \frac{5}{\cos\theta} = \frac{4}{\sin\theta}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{4}{5} \Rightarrow \tan\theta = \frac{4}{5}$$



Exercise 1

The Sine rule

Answer each of the following questions

1. XYZ is a triangle in which m (∠ X) = 80° , m (∠ Y) = 60° and z = 10 cm.
, find each of X and y to the nearest cm. « 15 cm. , 13 cm. »

$m(\angle Z) = 180 - (80 + 60) = 40°$

$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z} \Rightarrow \frac{x}{\sin 80} = \frac{y}{\sin 60} = \frac{10}{\sin 40}$

$x = \frac{10 \sin 80}{\sin 40} \simeq 15$ cm

$y = \frac{10 \sin 60}{\sin 40} \simeq 13$ cm

2 LMN is a triangle in which m = 68.4 cm. , m (∠ M) = 100° and m (∠ N) = 40°

, find : (1) ℓ

(2) The length of the radius of the circumcircle of the triangle LNM

(3) The area of the triangle LMN « 44.64 cm. , 34.73 cm. , 981.34 cm² »

$$m(\angle L) = 180 - [100 + 40] = 40^\circ$$

$$\frac{\ell}{\sin L} = \frac{m}{\sin M} = 2r$$

$$\frac{\ell}{\sin 40^\circ} = \frac{68.4}{\sin 100}$$

$$\ell = \frac{68.4 \sin 40}{\sin 100} \simeq 44.64 \text{ cm}$$

$$r = \frac{m}{2 \sin M} = \frac{68.4}{2 \sin 100} \simeq 34.73 \text{ cm}$$

$$\text{A. of } \Delta LMN = \frac{1}{2} \ell M \sin N$$

$$= \frac{1}{2} (44.64)(68.4) \sin 40^\circ$$

$$= 981.34 \text{ cm}^2$$

3 ABC is a triangle in which c = 4.5 cm. , m (∠ A) = 100° and m (∠ B) = 15°

Find the length of the smallest side of Δ ABC « 1.3 cm. »

$m(\angle C) = 180 - [100^\circ + 15^\circ] = 65^\circ$

Smallest side opposite to smallest angle which is $\Rightarrow b$

$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{b}{\sin 15} = \frac{4.5}{\sin 65}$$

$$b = \frac{4.5 \sin 15}{\sin 65} \simeq 1.3 \text{ cm.}$$

4. ABC is a triangle in which m ($\angle$ A) = 60° and a = $7\sqrt{3}$ cm. Find the area and the circumference of the circumcircle of Δ ABC $\left(\pi = \frac{22}{7}\right)$ « 154 cm^2, 44 cm. »

$$\frac{a}{\sin A} = 2r \Longrightarrow r = \frac{a}{2\sin A}$$

$$\therefore r = \frac{7\sqrt{3}}{2\sin 60} = 7$$

Area of the Circle $= \pi r^2$

$$= \frac{22}{7}(7)^2 = 154 \text{ cm}^2$$

Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 7$

$$= 44 \text{ cm}$$

(5) ABC is a triangle in which : a = 13 cm. , m (∠ A) = 53° 8̀ , c = 15 cm. Find the radius length of the circumcircle of Δ ABC , then find m (∠ C) « 8.1 cm. , 67° 23̀ 9̀̀ or 112° 36̀ 51̀̀ »

$$r = \frac{a}{2 \sin A} = \frac{13}{2 \sin 53^\circ 8'} \simeq 8.1 \text{ cm.}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{13}{\sin 53^\circ 8'} = \frac{15}{\sin C}$$

$$\sin C = \frac{15 \sin 53^\circ 8'}{13} \simeq 0.9231$$

1st : θ

2nd $\rightarrow$ $180 - \theta$

$$\theta = \sin^{-1}(0.9231) = 67^\circ\ 23'\ 9''$$

$$180^\circ - \theta^\circ = 112^\circ\ 36'\ 51''$$

6 Find the perimeter of the triangle ABC in which c = 8.7 cm. , m (∠ A) = 57° 13̀
and m (∠ B) = 64° 18̀ « 26.5 cm. »

$m(\angle C) = 180 - (57^\circ 13' + 64^\circ 18') = 58^\circ 29'$

$$\frac{c}{\sin C} = \frac{\text{P. of } \triangle ABC}{\sin A + \sin B + \sin C}$$

$$\frac{8.7}{\sin 58^\circ 29'} = \frac{\text{P. of } \triangle ABC}{\sin 57^\circ 13' + \sin 68^\circ 18' + \sin 58^\circ 29'}$$

P. of $\triangle ABC \simeq 26.76$ cm

7 ABC is an isosceles triangle in which : $a = b$ and $m(\angle A) = 15°$ and the perimeter of ΔABC is 25 cm. Find the area of the circumcircle of ΔABC « 474 cm^2 »

$\because a = b \quad \therefore m(\angle A) = m(\angle B) = 15°$

$m(\angle C) = 180 - (15 + 15) = 150°$

$$\frac{P. \text{ of } \Delta ABC}{\sin A + \sin B + \sin C} = 2r$$

$$\frac{25}{2(\sin 15 + \sin 15 + \sin 150)} = r$$

$r = 12.28$ cm

$A = \pi r^2 = \pi (12.28)^2$

$\simeq 474$ cm^2

8. XYZ is a triangle in which $\sin X + \sin Y + \sin Z = 2.37$ and its perimeter is 56.88 cm.

Find the length of the radius of the circumcircle of ΔXYZ « 12 cm. »

$$r = \frac{\text{P. of } \Delta XYZ}{2(\sin X + \sin Y + \sin Z)} = \frac{56.88}{2(2.37)}$$

$$\boxed{r = 12 \text{ cm}}$$

9. ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 1 : 3 : 5

Find the length of the smallest side of Δ ABC if its perimeter equals 16 cm. « 2.5 cm. »

A : B : C : Sum

1 : 3 : 5 : 9

? : ? : ? : 180°

$m(\angle A) = 20°$, $m(\angle B) = 60°$, $m(\angle C) = 100°$

Smallest Side is a

$$\frac{a}{\sin A} = \frac{P. \text{ of } \Delta ABC}{\sin A + \sin B + \sin C}$$

$$\frac{a}{\sin 20} = \frac{16}{\sin 20 + \sin 60 + \sin 100}$$

$$a = \frac{16 \sin 20}{\sin 20 + \sin 60 + \sin 100} \simeq 2.5 \text{ cm}$$

⑩ ABC is a triangle in which sin A : sin B : sin C = 2 : 4 : 5 and c – b = 3 cm.

Find each of a and b « 6 cm. , 12 cm. »

$$a : b : c = \sin A : \sin B : \sin C$$
$$= 2 : 4 : 5$$

$$a : b : c : c - b$$
$$2 : 4 : 5 : 1$$
$$? : ? : ? : 3$$

$$a = \frac{2 \times 3}{1} = 6$$

$$b = \frac{4 \times 3}{1} = 12$$

(11) ABC is a triangle in which m (∠ A) = $\frac{2}{3}$ m (∠ B) = $\frac{1}{2}$ m (∠ C) , the length of the radius of its circumcircle = 10 cm. Find the area of Δ ABC « 110 cm² »

$A + B + C = 180^\circ$

$A + \frac{3}{2}A + 2A = 180$

$4\frac{1}{2}A = 180 \Rightarrow m(\angle A) = 40^\circ$

$m(\angle B) = 60^\circ$, $m(\angle C) = 80^\circ$

$\frac{a}{\sin A} = \frac{b}{\sin B} = 2r$

$\frac{a}{\sin 40} = \frac{b}{\sin 60} = 20$

$a = 20 \sin 40^\circ \simeq 12.86$ cm

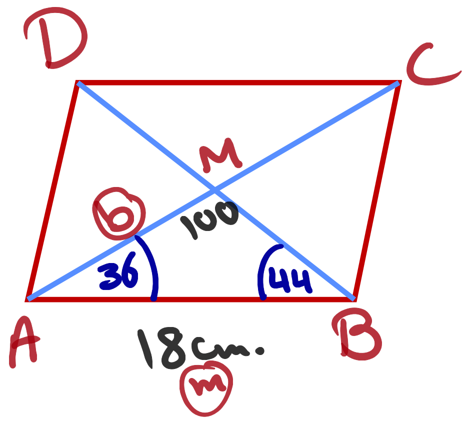
$b = 20 \sin 60^\circ \simeq 17.32$ cm

A. of Δ ABC $= \frac{1}{2} ab \sin C$

$= \frac{1}{2}(12.86)(17.32) \sin 80$

$\simeq 110$ cm²

(12) ABCD is a parallelogram in which AB = 18 cm. , m (∠ CAB) = 36° and m (∠ DBA) = 44°
Find the length of the diagonal $\overline{AC}$ and the area of the parallelogram. « 25.39 cm. , 269 cm^2. »



In $\triangle$ ABM

$m(\angle M) = 180 - [36 + 44]$
$= 100°$

$\frac{b}{\sin B} = \frac{m}{\sin M}$

$\frac{AM}{\sin 44} = \frac{18}{\sin 100}$

$AM = \frac{18 \sin 44}{\sin 100} \simeq 12.7$ cm

$AC = 2AM = 2(12.7) = 25.39$ cm

A. of ▱ABCD = 4 A. of $\triangle$ABM
$= 4 \times \frac{1}{2}\, b\, m \sin A$
$= 2(12.7)(18) \sin 36$

$\simeq 268.7$ cm^2

(13) ABCD is a trapezium in which $\overline{AD} \mathbin{//} \overline{BC}$, AD = 20 cm. , m (∠ D) = 120° , m (∠ B) = 62° and m (∠ ACB) = 23° 25̀

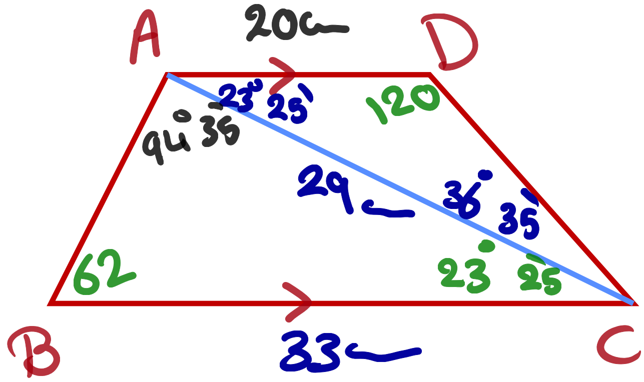
Find :

(1) The length of each of $\overline{AC}$ and $\overline{BC}$ to the nearest cm.

(2) The area of the trapezium ABCD to the nearest cm².

« 29 cm. , 33 cm. , 305 cm². »

In △ACD



$$\frac{c}{\sin C} = \frac{d}{\sin D}$$

$$\frac{20}{\sin 36^\circ 35'} = \frac{AC}{\sin 120}$$

$$AC = \frac{20 \sin 120^\circ}{\sin 36^\circ 35'} \simeq 29 \text{ cm}$$

In △ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{BC}{\sin 94^\circ 35'} = \frac{29}{\sin 62}$$

$$BC = \frac{29 \sin 94^\circ 35'}{\sin 62^\circ} \simeq 33 \text{ cm}$$

A. of trap. = A. of △ACD + A. of △ABC

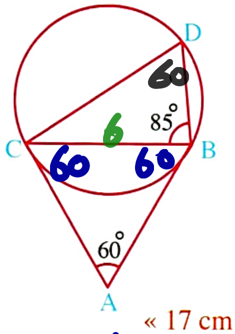
$$= \frac{1}{2} cd \sin A + \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(20)(29)\sin(23^\circ 25') + \frac{1}{2}(33)(29)\sin(23^\circ 25')$$

$$\simeq 305 \text{ cm}^2$$

(14) **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent segments to the circle at B and C , if m (∠ A) = 60° , m (∠ DBC) = 85° and the area of the triangle ABC = $9\sqrt{3}$ cm^2 , then find the perimeter of the triangle DBC to the nearest centimetre.



« 17 cm.»

∵ $\overline{AB}$ & $\overline{AC}$ are two tangat segments

∴ $AB = AC$, ∵ $m(\angle A) = 60°$

∴ $\triangle ABC$ is an equilateral $\triangle$

∴ A. of $\triangle ABC = \frac{1}{2} x^2 \sin 60 = 9\sqrt{3}$

$$\frac{\sqrt{3}}{4} x^2 = 9\sqrt{3}$$

$x^2 = 36 \Rightarrow x = 6$ cm

∴ $AB = AC = BC = 6$ cm

∵ $m(\angle BDC) = m(\angle ABC) = 60°$

[inscribed angle & angle of tangency sub. by $\overset{\frown}{BC}$]

∴ $m(\angle BCD) = 180° - [85 + 60] = 35°$

$$\frac{d}{\sin D} = \frac{P. \text{ of } \triangle BCD}{\sin B + \sin C + \sin D}$$

∴ P. of $\triangle BCD = \frac{6[\sin 85 + \sin 35 + \sin 60]}{\sin 60}$

$\simeq 17$ cm

15 **In the triangle ABC , prove that :**

$$\sin A + \sin B + \sin C = \frac{4\,S\,\Delta}{a\,b\,c}$$

where S is half of the triangle's perimeter and Δ is the triangle's area.

$$R.H.S. = \frac{4S\Delta}{abc} = \frac{\cancel{4} \times \cancel{\tfrac{1}{2}}(a+b+c) \times \cancel{\tfrac{1}{2}}\cancel{a}\cancel{b}\sin C}{\cancel{a}\cancel{b}c}$$

$$= (a+b+c) \times \frac{\sin C}{c}$$

$$= \cancel{(a+b+c)} \times \frac{\sin A + \sin B + \sin C}{\cancel{a+b+c}}$$

$$= \sin A + \sin B + \sin C$$

$$= L.H.S$$