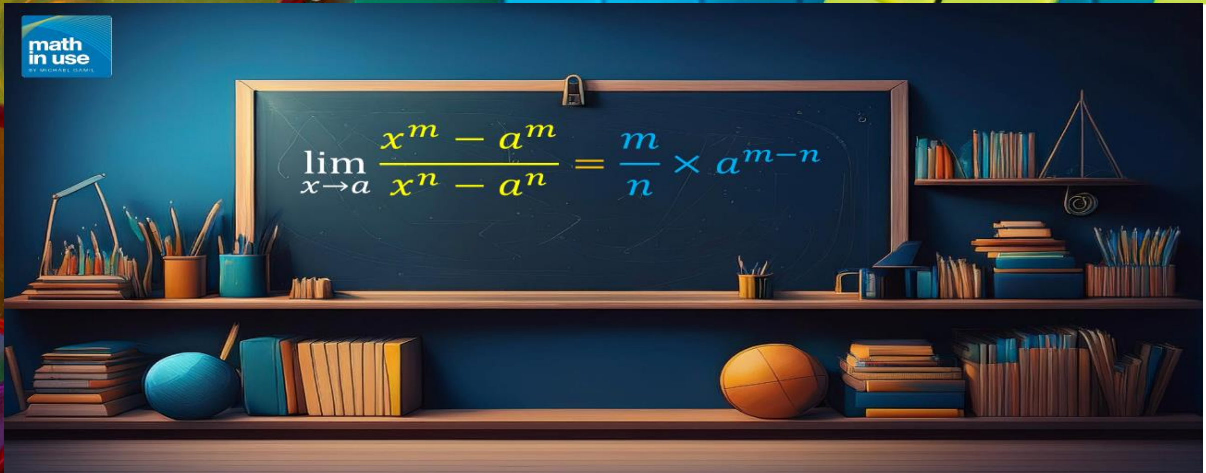

Finding limit of the function By theorem (4)

Choose the correct answer

$\lim_{x \to a} \frac{x^n - a^n}{x^m - a^m} =$ $\frac{n}{m}(a)^{n-m}$ ....

(a) $\frac{m}{n}$

(b) $\frac{m}{n}(a)^{m-n}$

(c) $\frac{n}{m}(a)^{m-n}$

(d) $\frac{n}{m}(a)^{n-m}$

$\lim\limits_{y \to 2} \dfrac{y^5 - 32}{y - 2} = \dots\dots\dots\dots$

(a) $31\, y^4$ (b) 32×2^4 (c) 64 (d) 5×2^4

$$\lim_{y \to 2} \frac{y^5 - 2^5}{y^1 - 2^1} = \frac{5}{1}(2)^4$$

$$= 5 \times 2^4$$

$\lim\limits_{x \to 1} \frac{x^{10} - 1}{x - 1} = \ldots\ldots\ldots\ldots\ldots$

(a) zero. (b) 1 (c) 9 (d) 10

$$\lim_{x \to 1} \frac{x^{10} - 1^{10}}{x^1 - 1^1} = \frac{10}{1}(1)^9 = 10$$

$\lim_{x \to 2} \frac{x^{-1} - 2^{-1}}{x^{-4} - 2^{-4}} = \dots\dots\dots\dots$

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{32}$ (d) 8

$= \frac{-1}{-4} \times (2)^{-1+4} = \frac{1}{4} \times 2^{3} = 2$

$\lim_{x \to -1} \frac{x^5+1}{x+1} = \ldots\ldots\ldots\ldots\ldots$

(a) 5 (b) 4 (c) – 5 (d) – 4

$$\lim_{x \to -1} \frac{x^5-(-1)^5}{x^1-(-1)^1} = \frac{5}{1} \times (-1)^4$$

$$= 5$$

$\lim\limits_{x \to 2} \dfrac{x^5 - 32}{x^3 - 8} = \dots\dots\dots\dots$

(a) 4 (b) $\frac{5}{3}$ (c) zero (d) $6\frac{2}{3}$

$$\lim_{x \to 2} \frac{x^5 - (2)^5}{x^3 - (2)^3} = \frac{5}{3} \times (2)^2$$

$$= \frac{5}{3} \times 4 = \frac{20}{3}$$

$$= 6\,^{2}/_{3}$$

$$\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{32x^5 - 1} = \ldots\ldots\ldots\ldots$$

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{2}{9}$ (d) $\frac{1}{8}$

$x \to \frac{1}{2}$

$2x \to 1$

$$\lim_{x \to \frac{1}{2}} \frac{4(x^2 - \frac{1}{4})}{32(x^5 - \frac{1}{32})}$$

$$\lim_{x \to \frac{1}{2}} \frac{1}{8} \times \lim_{x \to \frac{1}{2}} \frac{x^2 - (\frac{1}{2})^2}{x^5 - (\frac{1}{2})^5}$$

$$\frac{1}{8} \times \frac{2}{5} \times (\frac{1}{2})^{-3}$$

$$= 2/5$$

Another Sol.

$$\lim_{2x \to 1} \frac{(2x)^2 - (1)^2}{(2x)^5 - (1)^5}$$

$$= \frac{2}{5} \times (1)^{-3} = \frac{2}{5}$$

$$\lim_{x \to -2} \frac{x^7 + 128}{x^4 - 16} = \dots\dots\dots\dots\dots$$

(a) 9 (b) -9 (c) -14 (d) 14

$$\lim_{x \to -2} \frac{x^7 - (-2)^7}{x^4 - (-2)^4} = \frac{7}{4} \times (-2)^3$$

$$= -14$$

$\lim\limits_{x \to 16} \dfrac{\sqrt[4]{x^5} - 32}{x - 16} = \dots\dots\dots\dots$

$\sqrt[n]{x^m} = x^{m/n}$

(a) 5 (b) $\frac{5}{2}$ (c) $\frac{5}{4}$ (d) $\frac{5}{8}$

$$\lim_{x \to 16} \frac{x^{\frac{5}{4}} - (16)^{\frac{5}{4}}}{x^1 - 16^1} = \frac{5/4}{1} (16)^{\frac{5}{4} - 1}$$

$$= \frac{5}{4} \times 16^{\frac{1}{4}} = \frac{5}{2}$$

$\lim\limits_{h \to 0} \dfrac{(x+h)^7 - x^7}{h} = \ldots\ldots\ldots\ldots$

$h \to 0$

$x+h \to x$

(a) x^7 (b) $7x^6$ (c) zero. (d) 1

$$\lim_{(x+h)\to x} \frac{(x+h)^7 - (x)^7}{(x+h)^1 - (x)^1} = \frac{7}{1}(x)^6$$

$$= 7x^6$$

$\lim_{h \to 0} \frac{(2-3h)^7 - 128}{4h} = \dots\dots\dots\dots$

(a) 336 (b) – 336 (c) 448 (d) – 448

$h \to 0$

$-3h \to 0$

$2-3h \to 2$

$\frac{-3}{4} \lim_{(2-3h) \to 2} \frac{(2-3h)^7 - (2)^7}{(2-3h)^1 - (2)^1}$

$-\frac{3}{4} \times \frac{7}{1} (2)^6 = -336$

$\lim\limits_{h \to 0} \dfrac{(3h-1)^5 + 1}{5h} = \ldots\ldots\ldots\ldots\ldots$

(a) -3 (b) $\frac{3}{5}$ (c) 3 (d) 5

$$\frac{3}{5} \lim_{(3h-1) \to -1} \frac{(3h-1)^5 - (-1)^5}{(3h-1)^1 - (-1)^1}$$

$$\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{1} \times (-1)^4 = 3$$

$\lim\limits_{x \to 1} \dfrac{x^6 - 64}{x - 2} = \dots\dots\dots\dots$

(a) $6\,\cancel{(2)^5}$ (b) 128 (c) $64\,(2)^5$ (d) 63

$\lim \dfrac{x^6 - 2^6}{x^1 - 2^1}$

$= \dfrac{6}{1}(2)^5$

$\dfrac{1 - 64}{1 - 2} = \dfrac{-63}{-1} = 63$

$$\lim_{x \to 1} \frac{x^{\frac{13}{2}} - x^{\frac{1}{2}}}{x^{\frac{7}{2}} - x^{\frac{1}{2}}} = \ldots\ldots\ldots\ldots$$

(a) $\frac{13}{7}$ (b) 1 (c) 2 (d) x

$$\lim_{x \to 1} \frac{\cancel{x^{\frac{1}{2}}}(x^6 - 1)}{\cancel{x^{\frac{1}{2}}}(x^3 - 1)}$$

$$\frac{x^{\frac{13}{2}}}{x^{\frac{1}{2}}} = x^6$$

$$\frac{x^{\frac{7}{2}}}{x^{\frac{1}{2}}} = x^3$$

$$\lim_{x \to 1} \frac{x^6 - 1^6}{x^3 - 1^3} = \frac{6}{3}(1)^3 = 2$$

$$\lim_{x \longrightarrow 1} \frac{x^2 - x^{-2}}{x - x^{-1}} = \dots\dots\dots\dots$$

(a) zero (b) 1 (c) 2 (d) – 2

$$\lim_{x\to 1} \frac{x^{-2}(x^4 - 1)}{x^{-1}(x^2 - 1)}$$

$$\frac{x^2}{x^{-2}} = x^4$$

$$\frac{x^1}{x^{-1}} = x^2$$

$$\lim_{x\to 1} x^{-1} \times \lim_{x\to 1} \frac{x^4 - 1^4}{x^2 - 1^2}$$

$$1 \times \frac{4}{2}(1)^2 = 2$$

$$\lim_{x \to 0} \frac{(x+2)^5 - 32}{x} = \ldots\ldots\ldots\ldots\ldots$$

(a) 25 (b) 64 (c) 80 (d) 100

$$\lim_{(x+2) \to 2} \frac{(x+2)^5 - (2)^5}{(x+2)^1 - (2)^1} = \frac{5}{1} (2)^4$$

$$= 80$$

$\lim\limits_{x \to 5} \frac{(x-3)^7 - 128}{x-5} = \dots\dots\dots\dots$

(a) 7 (b) 28 (c) 64 (d) 448

$\lim \frac{(x-3)^7 - (2)^7}{(x-3) - 5 + 3}$

$\lim\limits_{(x-3) \to 2} \frac{(x-3)^7 - (2)^7}{(x-3)^1 - (2)^1} = \frac{7}{1}(2)^6$

$= 448$

$\lim_{x \to 2} \frac{(x+1)^4 - 81}{x-2} = \ldots\ldots\ldots\ldots$

(a) 18 (b) 81 (c) – 108 (d) 108

$$\lim_{x \to 2} \frac{(x+1)^4 - (3)^4}{(x+1) - 2 - 1}$$

$$\lim_{(x+1) \to 3} \frac{(x+1)^4 - (3)^4}{(x+1)^1 - (3)^1} = \frac{4}{1}(3)^3 = 108$$

$$\lim_{x \to 1} \frac{x^a - x^c}{x^a - x^d} = \ldots\ldots\ldots\ldots$$

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(a) $\frac{a+c}{a+d}$ (b) $a\,x^{c-d}$ (c) $\frac{a-c}{a-d}$ (d) $\frac{c}{d}$

$$\lim_{x \to 1} \frac{x^c (x^{a-c} - 1)}{x^d (x^{a-d} - 1)}$$

$$\frac{x^a}{x^c} = x^{a-c}$$

$$\frac{x^a}{x^d} = x^{a-d}$$

$$\lim_{x \to 1} \frac{x^c}{x^d} \times \lim_{x \to 1} \frac{x^{a-c} - 1^{a-c}}{x^{a-d} - 1^{a-d}}$$

$$\frac{1^c}{1^d} \times \boxed{\frac{a-c}{a-d}} \times (1)^{a-c-a+d}$$

$$= \frac{a-c}{a-d}$$

$$\lim_{x \to 1} \frac{1-\sqrt[n]{x}}{1-\sqrt[m]{x}} = \ldots\ldots\ldots\ldots$$

$\frac{1}{n} \div \frac{1}{m} = \frac{1}{n} \times \frac{m}{1} = \frac{m}{n}$

(a) 1 (b) $\frac{n}{m}$ (c) -1 (d) $\frac{m}{n}$

$$\lim_{x \to 1} \frac{\cancel{-}\left(x^{\frac{1}{n}} - 1^{\frac{1}{n}}\right)}{\cancel{-}\left(x^{\frac{1}{m}} - 1^{\frac{1}{m}}\right)} = \frac{1/n}{1/m} (1)^{\frac{1}{n}-\frac{1}{m}}$$

$$= \frac{m}{n}$$

$\lim\limits_{x \to 9} \dfrac{3-\sqrt{x}}{27-\sqrt{x^3}} = \dots\dots\dots\dots$

(a) $\frac{1}{9}$ (b) $\frac{1}{27}$ (c) 3 (d) $-\frac{1}{27}$

$$\lim_{x \to 9} \frac{\cancel{-}(x^{\frac{1}{2}} - 3)}{\cancel{-}(x^{\frac{3}{2}} - 27)} = \lim_{x \to 9} \frac{x^{\frac{1}{2}} - (9)^{\frac{1}{2}}}{x^{\frac{3}{2}} - (9)^{\frac{3}{2}}}$$

$$\frac{1/2}{3/2} (9)^{\frac{1}{2}-\frac{3}{2}} = \frac{1}{3} \times \frac{1}{9} = \frac{1}{27}$$

$\lim\limits_{x \to -2} \dfrac{(x+3)^5 - 1}{x^2 - 4} = \ldots\ldots\ldots\ldots$

$\boxed{\dfrac{a}{bc} = \dfrac{a}{b} \times \dfrac{1}{c}}$ or $\dfrac{a}{1} \times \dfrac{b}{c}$

(a) $\frac{5}{4}$ (b) $\frac{-5}{4}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{4}$

$$\lim_{x \to -2} \frac{(x+3)^5 - 1}{(x+2)(x-2)}$$

$$\lim_{x \to -2} \frac{(x+3)^5 - (1)^5}{x+2} \times \lim_{x \to -2} \frac{1}{x-2}$$

$$\lim_{x \to -2} \frac{(x+3)^5 - (1)^5}{(x+3)^1 - (1)^1} \times \lim_{x \to -2} \frac{1}{x-2}$$

$$\frac{5}{1}(1)^4 \times \frac{1}{-2-2} = -\frac{5}{4}$$

$$\lim_{x \to 2} \frac{x^5 - 32}{x^2 + 3x - 10} = \ldots\ldots\ldots\ldots$$

(a) 80 (b) $\frac{80}{7}$ (c) $\frac{7}{80}$ (d) $\frac{1}{80}$

$$\lim_{x \to 2} \frac{x^5 - 32}{(x-2)(x+5)}$$

$$\lim_{x \to 2} \frac{x^5 - 2^5}{x^1 - 2^1} \times \lim_{x \to 2} \frac{1}{x+5}$$

$$\frac{5}{1}(2)^4 \times \frac{1}{7} = \frac{80}{7}$$

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$\lim_{x \to 1} \frac{\sqrt[5]{x} + 2\sqrt{x} - 3}{x - 1} = \dots\dots\dots\dots$

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(a) $\frac{6}{5}$ (b) 2 (c) 5 (d) 3

$\boxed{\frac{a+b}{c} \quad \frac{a}{c} + \frac{b}{c}}$

$$\lim_{x \to 1} \frac{\sqrt[5]{x} - 1 + 2\sqrt{x} - 2}{x - 1}$$

$$\lim_{x \to 1} \frac{x^{\frac{1}{5}} - 1^{\frac{1}{5}}}{x^1 - 1^1} + 2\lim_{x \to 1} \frac{x^{\frac{1}{2}} - 1^{\frac{1}{2}}}{x^1 - 1^1}$$

$$\frac{1}{5}(1) + \cancel{2} \times \frac{1}{\cancel{2}}(1)$$

$$\frac{1}{5} + 1 = 1\frac{1}{5} = \frac{6}{5}$$

If $f(x) = x^5$, $g(x) = x^2 - 4$, then $\lim_{x \to 2} \frac{f(x) - 32}{g(x)} = \dots\dots\dots\dots$

(a) – 20 (b) 20 (c) ± 20 (d) 32

$$\lim_{x \to 2} \frac{x^5 - 32}{x^2 - 4} = \lim_{x \to 2} \frac{x^5 - 2^5}{x^2 - 2^2}$$

$$= \frac{5}{2}(2)^3 = 20$$

If $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x^m - a^m} = \dfrac{n}{m}$, then a = ……………

(a) 1 (b) n (c) m (d) $\frac{m}{n}$

$$\frac{n}{m}(a)^{n-m} = \frac{n}{m}$$

$$\boxed{\therefore a = 1}$$

If $\lim\limits_{x \to 2} \frac{(x)^n - (2)^n}{x - 2} = 32$, then n = ...

(a) 3
(b) 4
(c) 9
(d) 12

$\frac{x^4 - 2^4}{x^1 - 2^1} = 32$

$4(2)^3 = 32$

$\frac{n}{1}(2)^{n-1} = 32$

$4(2)^3 = 2^5 = 32$

If $\lim\limits_{x \to a} \dfrac{x^8 - a^8}{x^6 - a^6} = 48$, then a = ...

(a) 4 (b) 6 (c) ± 4 (d) ± 6

$$\frac{8}{6}(a)^2 = 48$$

$$a^2 = 48 \div \frac{4}{3}$$

$$a^2 = 36 \implies \boxed{a = \pm 6}$$

$\lim_{x \to 5} \frac{\sqrt[3]{x+3}-2}{x-5} = \dots\dots\dots\dots$

(a) $\frac{1}{12}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) 1

$$\lim_{x \to 5} \frac{(x+3)^{\frac{1}{3}}-(8)^{\frac{1}{3}}}{(x+3)-5-3}$$

$$\lim_{x \to 5} \frac{(x+3)^{\frac{1}{3}}-(8)^{\frac{1}{3}}}{(x+3)^{1}-(8)^{1}}$$

$$= \frac{1}{3}(8)^{\frac{1}{3}-1} = \frac{1}{12}$$

$$\lim_{x \to 0} \frac{(1+x)^{\frac{5}{2}} - 1}{x} = \ldots\ldots\ldots\ldots$$

(a) $\frac{5}{2}$ (b) zero. (c) $\frac{2}{5}$ (d) does not exist.

$$\lim_{(1+x) \to 1} \frac{(1+x)^{\frac{5}{2}} - (1)^{\frac{5}{2}}}{(1+x)^{1} - (1)^{1}} = \frac{5}{2}(1)^{\frac{5}{2}-1}$$

$$= \frac{5}{2}$$

$$\lim_{x \to 0} \frac{(x+2)^6 - 64}{x^2 + 16x} = \ldots\ldots\ldots\ldots$$

(a) 6 (b) 12 (c) 16 (d) 64

$$\lim_{x \to 0} \frac{(x+2)^6 - 64}{x(x+16)}$$

$$\lim_{x \to 0} \frac{1}{x+16} \times \lim_{(x+2) \to 2} \frac{(x+2)^6 - 2^6}{(x+2) - 2}$$

$$\frac{1}{16} \times \frac{6}{1} \cdot (2)^5 = 12$$

$\lim\limits_{x \to 1} \left(\dfrac{x^6 - x^7 + x^8 - x^9}{x - 1} \right) = \ldots\ldots\ldots\ldots\ldots$

(a) 30 (b) -2 (c) 3 (d) 9

$\dfrac{a+b}{c}$

$\dfrac{a}{c} + \dfrac{b}{c}$

$$\lim_{x \to 1} \frac{(x^6 - x^7) + (x^8 - x^9)}{x - 1}$$

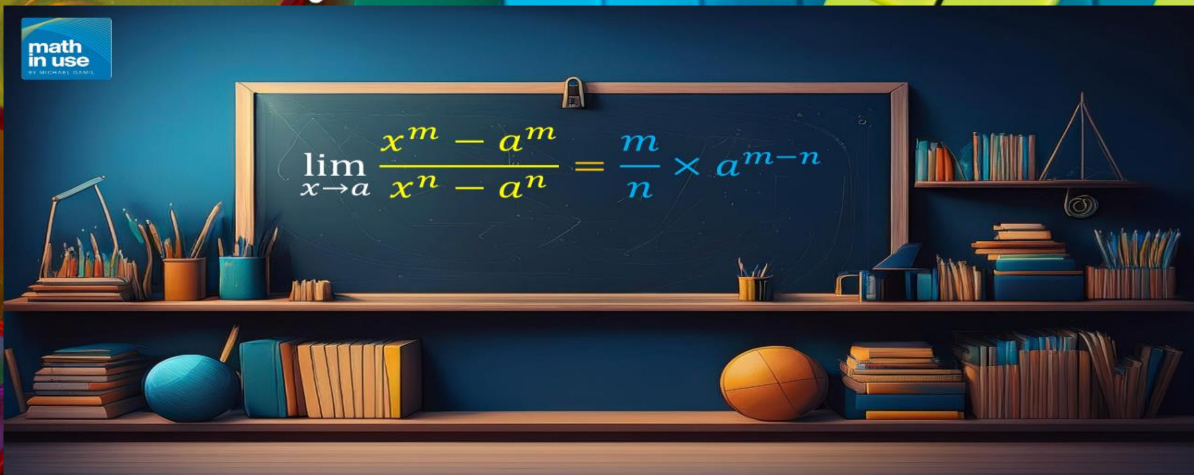
$$\lim_{x \to 1} \frac{x^6 - x^7}{x - 1} + \lim_{x \to 1} \frac{x^8 - x^9}{x - 1}$$

$$\lim_{x \to 1} \frac{-x^6 \cancel{(x-1)}}{\cancel{x-1}} + \lim_{x \to 1} \frac{-x^8 \cancel{(x-1)}}{\cancel{x-1}}$$

$$\lim_{x \to 1} -x^6 + \lim_{x \to 1} -x^8$$

$$(-1) + (-1)$$

$$= -2$$



Finding limit of the function By theorem (4)

Answer each of the following questions

① **Find each of the following**

$\boxed{1}$ $\lim\limits_{x \to 2} \frac{x^3 - 8}{x - 2}$ $= \lim\limits_{x \to 2} \frac{x^3 - 2^3}{x - 2}$

$= \frac{3}{1} (2)^2 = 12$

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2 $\lim\limits_{x \to a} \dfrac{x^5 - a^5}{x - a} = \dfrac{5}{1}(a)^4$

$= 5a^4$

3 $\lim_{x \to 2} \frac{x^6 - 64}{x^7 - 128}$

$$\lim_{x \to 2} \frac{x^6 - 2^6}{x^7 - 2^7} = \frac{6}{7}(2)^{-1}$$

$$= \frac{3}{7}$$

$\boxed{4}$ $\lim\limits_{x \to -2} \dfrac{x^6 - 64}{3x + 6}$ $= \lim\limits_{x \to -2} \dfrac{x^6 - 64}{3(x+2)}$

$$\lim_{x \to -2} \frac{1}{3} \times \lim_{x \to -2} \frac{x^6 - (-2)^6}{x^1 - (-2)^1}$$

$$\frac{1}{3} \times \frac{6}{1}(-2)^5 = -64$$

5 $\lim\limits_{x \longrightarrow 1} \dfrac{1 - x^9}{x^7 - 1} = \lim\limits_{x \to 1} \dfrac{-(x^9 - 1^9)}{x^7 - 1^7}$

$-1 \times \lim\limits_{x \to 1} \dfrac{x^9 - 1^9}{x^7 - 1^7}$

$-1 \times \dfrac{9}{7} (1)^2 = -\dfrac{9}{7}$

6 $\lim\limits_{x \to \frac{-2}{3}} \frac{243x^5 + 32}{27x^3 + 8}$

$$\lim_{x \to \frac{-2}{3}} \frac{243\left(x^5 + \frac{32}{243}\right)}{27\left(x^3 + \frac{8}{27}\right)}$$

$$\lim_{x \to \frac{-2}{3}} \frac{243}{27} \times \lim_{x \to \frac{-2}{3}} \frac{x^5 - \left(-\frac{2}{3}\right)^5}{x^3 - \left(-\frac{2}{3}\right)^3}$$

$$9 \times \frac{5}{3} \times \left(-\frac{2}{3}\right)^2 = \frac{20}{3}$$

$\boxed{7}$ $\lim\limits_{x \to 2} \dfrac{x^{-7} - (2)^{-7}}{x - 2} = \dfrac{-7}{1} \times (2)^{-7-1}$

$= -7(2)^{-8} =$

$= \dfrac{-7}{256}$

8 $\lim\limits_{x \to \sqrt{2}} \dfrac{x^7 - 8\sqrt{2}}{x^2 - 2}$ $= \lim\limits_{x \to \sqrt{2}} \dfrac{x^7 - (\sqrt{2})^7}{x^2 - (\sqrt{2})^2}$

$= \dfrac{7}{2}(\sqrt{2})^5 = \dfrac{7}{2}(\sqrt{2})^4 \cdot \sqrt{2}$

$= 14\sqrt{2}$

9 $\lim\limits_{x \to \frac{-3}{\sqrt{2}}} \dfrac{8x^6 - 729}{\sqrt{2}\,x + 3}$

$$\lim_{x \to \frac{-3}{\sqrt{2}}} \frac{8\left(x^6 - \frac{729}{8}\right)}{\sqrt{2}\left(x + \frac{3}{\sqrt{2}}\right)}$$

$$\lim_{x \to \frac{-3}{\sqrt{2}}} \frac{8}{\sqrt{2}} \times \lim_{x \to \frac{-3}{\sqrt{2}}} \frac{x^6 - \left(-\frac{3}{\sqrt{2}}\right)^6}{x^1 - \left(-\frac{3}{\sqrt{2}}\right)^1}$$

$$\frac{8}{\sqrt{2}} \times \frac{6}{1} \times \left(-\frac{3}{\sqrt{2}}\right)^5$$

$$= -1458$$

$\boxed{10}$ $\lim\limits_{x \to 2} \dfrac{x^{-5} - \frac{1}{32}}{x^{-7} - \frac{1}{128}}$

$= \lim\limits_{x \to 2} \dfrac{x^{-5} - (2)^{-5}}{x^{-7} - (2)^{-7}}$

$= \dfrac{\cancel{-}5}{\cancel{-}7} \times (2)^{2} = \dfrac{20}{7}$

11 $\lim_{x \to 2} \frac{x^{-8} - (16)^{-2}}{x - 2}$

$$\lim_{x \to 2} \frac{x^{-8} - (2)^{-8}}{x^{1} - 2^{1}} = \frac{-8}{1} (2)^{-9}$$

$$= -\frac{1}{64}$$

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12 $\lim\limits_{x \to 1} \dfrac{\sqrt[7]{x^{1}}-1}{x-1}$

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$$\lim_{x \to 1} \frac{x^{\frac{1}{7}} - 1^{\frac{1}{7}}}{x^1 - 1^1} = \frac{1/7}{1} (1)^{\frac{1}{7}-1}$$

$$= \frac{1}{7}$$

13 $\lim\limits_{x \to 16} \dfrac{\sqrt[4]{x^7} - 128}{x - 16}$

$= \lim\limits_{x \to 16} \dfrac{x^{\frac{7}{4}} - 16^{\frac{7}{4}}}{x^1 - 16^1}$

$= \frac{7}{4} (16)^{\frac{7}{4} - 1} = 14$

14 $\lim\limits_{x \to 1} \dfrac{x^{\frac{21}{2}} - x^{\frac{1}{2}}}{x^{\frac{14}{3}} - x^{\frac{2}{3}}}$

$\dfrac{x^{\frac{21}{2}}}{x^{\frac{1}{2}}} = x^{10}$

$\dfrac{x^{\frac{14}{3}}}{x^{\frac{2}{3}}} = x^{4}$

$\dfrac{x^{\frac{1}{2}}}{x^{\frac{2}{3}}}$

$$= \lim_{x \to 1} \frac{x^{\frac{1}{2}}(x^{10} - 1^{10})}{x^{\frac{2}{3}}(x^{4} - 1^{4})}$$

$$\lim_{x \to 1} x^{-\frac{1}{6}} \times \lim_{x \to 1} \frac{x^{10} - 1^{10}}{x^{4} - 1^{4}}$$

$$(1)^{-\frac{1}{6}} \times \frac{10}{4}(1)^{10-4} = \frac{5}{2}$$

15 $\lim\limits_{x \to 2} \dfrac{x^{10} - 1024}{x^2 - 3x + 2}$

$$\lim_{x \to 2} \frac{x^{10} - 1024}{(x-2)(x-1)}$$

$$\lim_{x \to 2} \frac{1}{x-1} \times \lim_{x \to 2} \frac{x^{10} - 2^{10}}{x^1 - 2^1}$$

$$\frac{1}{2-1} \times \frac{10}{1} (2)^9 = \underline{5120}$$

16 $\lim\limits_{x \to 0} \dfrac{(1+x)^{10} - 1^{10}}{(1+x)^{7} - 1^{7}}$

$$\lim_{(1+x) \to 1} \frac{(1+x)^{10} - 1^{10}}{(1+x)^{7} - 1^{7}} = \frac{10}{7} \times (1)^{3}$$

$$= \frac{10}{7}$$

17 $\lim\limits_{x \to -2} \dfrac{(x+3)^5 - 1}{x + 2 \; +1 - 1}$

$$\lim\limits_{(x+3) \to 1} \frac{(x+3)^5 - (1)^5}{(x+3)^1 - (1)^1}$$

$$= \frac{5}{1}(1)^4 = 5$$

$\boxed{18}$ $\lim\limits_{h \to 0} \dfrac{(3+h)^4 - 81}{6h}$

$\frac{1}{6} \lim\limits_{(3+h) \to 3} \dfrac{(3+h)^4 - (3)^4}{(3+h)^1 - (3)^1}$

$\frac{1}{6} \times \frac{4}{1} \times (3)^3 = 18$

$\boxed{19}$ $\lim_{x \to 0} \dfrac{(1-2x)^5 - 1}{5x}$

$$\frac{-2}{5} \lim_{(1-2x)\to 1} \frac{(1-2x)^5 - (1)^5}{(1-2x)^1 - (1)^1}$$

$$-\frac{2}{\cancel{5}} \times \frac{\cancel{5}}{1} \times (1)^4 = -2$$

20 $\lim\limits_{h \to 0} \dfrac{(x-2h)^{17} - x^{17}}{51\,h}$

$$\frac{-2}{51} \times \lim_{(x-2h) \to x} \frac{(x-2h)^{17} - x^{17}}{(x-2h)^{1} - x^{1}}$$

$$-\frac{2}{51} \times \frac{17}{1} \times x^{16} = -\frac{2}{3} x^{16}$$

21 $\lim\limits_{x \to 0} \dfrac{\sqrt[3]{1+3x}-1}{2x}$

$$\frac{3}{2} \lim_{(1+3x)\to 1} \frac{(1+3x)^{\frac{1}{3}} - (1)^{\frac{1}{3}}}{(1+3x)^{1} - (1)^{1}}$$

$$\frac{\cancel{3}}{2} \times \frac{1}{\cancel{3}} \times (1)^{\frac{1}{3}-1} = \frac{1}{2}$$

22 $\lim\limits_{x \to 7} \dfrac{\sqrt[5]{x+25}-2}{x-7}$

$$\lim_{x \to 7} \frac{(x+25)^{\frac{1}{5}} - 2}{(x+25)-7-25}$$

$$\lim_{(x+25) \to 32} \frac{(x+25)^{\frac{1}{5}} - (32)^{\frac{1}{5}}}{(x+25)^{1} - (32)}$$

$$\frac{1}{5}(32)^{\frac{1}{5}-1} = \frac{1}{80}$$

23 $\lim_{x \to 2} \frac{x^5 + x^2 - 36}{x - 2}$

$\frac{a+b}{c}$

$\boxed{\frac{a}{c} + \frac{b}{c}}$

$\lim_{x \to 2} \frac{(x^5 - 2^5) + (x^2 - 2^2)}{x - 2}$

$\lim_{x \to 2} \frac{x^5 - 2^5}{x^1 - 2^1} + \lim_{x \to 2} \frac{x^2 - 2^2}{x^1 - 2^1}$

$\frac{5}{1}(2)^4 + \frac{2}{1}(2)^1$

$= 84$

24 $\lim\limits_{x \to 2} \dfrac{x^5 + x^2 - 36}{x - 2}$

② **Find the value of a if :** $\lim\limits_{x \to a} \dfrac{x^{12} - a^{12}}{x^{10} - a^{10}} = 30$

$$\frac{12}{10}(a)^2 = 30$$

$$\frac{6}{5}a^2 = 30$$

$$a^2 = 30 \div \frac{6}{5}$$

$$a^2 = 25$$

$$a = \pm\sqrt{25}$$

$$\boxed{a = \pm 5}$$

③ **Find the value of k if :** $\lim_{x \to -1} \frac{x^{15}+1}{x+1} = \lim_{x \to k} \frac{x^5-k^5}{x^3-k^3}$

$$\lim_{x \to -1} \frac{x^{15}-(-1)^{15}}{x^1-(-1)^1} = \lim_{x \to k} \frac{x^5-k^5}{x^3-k^3}$$

$$\frac{15}{1}(-1)^{14} = \frac{5}{3}(k)^2$$

$$\frac{5}{3}k^2 = 15$$

$$k^2 = 9$$

$$k = \pm\sqrt{9}$$

$$k = \pm 3$$

4. **Find the value of n and ℓ if :** If $\lim\limits_{x \to 2} \dfrac{x^n - 64}{x - 2} = \ell$

$$\lim_{x \to 2} \frac{x^n - 64}{x - 2} = \ell$$

$$\lim_{x \to 2} \frac{x^6 - (2)^6}{x^1 - 2^1} = \ell$$

$$\frac{6}{1}(2)^5 = \ell = 192$$

$\therefore n = 6$ $\quad$ $\ell = 192$

⑤ **Find each of the following**

1 $\lim\limits_{x \to 2} \left(\frac{x^3-8}{x^5-32} + \frac{x^4-16}{x^7-128} \right)$

$$\lim_{x \to 2} \frac{x^3-2^3}{x^5-2^5} + \lim_{x \to 2} \frac{x^4-2^4}{x^7-2^7}$$

$$\frac{3}{5}(2)^{3-5} + \frac{4}{7}(2)^{4-7} = \frac{31}{140}$$

⑤ **Find each of the following**

2 $\lim\limits_{x \to -3} \left(\dfrac{x^4 - 81}{x^3 + 27}\right)^3$

$$\lim_{x \to -3} \left(\frac{x^4 - (-3)^4}{x^3 - (-3)^3}\right)^3$$

$$\left[\frac{4}{3} \times (-3)^1\right]^3 = (-4)^3$$

$$= -64$$

⑤ Find each of the following

3 $\lim\limits_{x \to 1} \frac{x^{12} - 2x^6 + 1}{x^2 - 2x + 1} = \lim\limits_{x \to 1} \frac{(x^6 - 1)^2}{(x - 1)^2}$

$= \lim\limits_{x \to 1} \left(\frac{x^6 - 1^6}{x^1 - 1^1} \right)^2$

$= \left[\frac{6}{1} (1)^5 \right]^2 = (6)^2$

$= 36$

⑤ **Find each of the following**

$$\frac{a}{c}\Big|\frac{b}{d} = \frac{a}{c} \times \frac{b}{d}$$

4 $\lim\limits_{x \to 1} \frac{\left(\sqrt[3]{x}-1\right)\left(\sqrt[5]{x^2}-1\right)}{(x-1)^2}$

$$\lim_{x \to 1} \frac{\sqrt[3]{x}-1}{x-1} \times \lim_{x \to 1} \frac{\sqrt[5]{x^2}-1}{x-1}$$

$$\lim_{x \to 1} \frac{x^{\frac{1}{3}}-1^{\frac{1}{3}}}{x^1-1^1} \times \lim_{x \to 1} \frac{x^{\frac{2}{5}}-1^{\frac{2}{5}}}{x^1-1^1}$$

$$\frac{1}{3}(1) \times \frac{2}{5}(1)$$

$$\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$$