Exercise 2

The moment of a force about a point in 2D-coordinate system

First : Moment by using vector product

Choose the correct answer

If the force $\vec{F}$ acts at the point (A) , $\overrightarrow{M_O}$ is the moment of $\vec{F}$ about the origin O , then ....

(a) $\overrightarrow{M_O} = \overrightarrow{OA} \times \vec{F}$

(b) $\overrightarrow{M_O} = \overrightarrow{OA} \cdot \vec{F}$

(c) $\overrightarrow{M_O} = \vec{F} \cdot \overrightarrow{OA}$

(d) $\overrightarrow{M_O} = \vec{F} \times \overrightarrow{OA}$

If $\vec{F} = 2\hat{i} - 3\hat{j}$, A (2 , 1) $\in$ the line of action of $\vec{F}$, O is the origin , then $\vec{M}_O = \cdots\cdots\cdots\cdots\cdots\; \hat{k}$

(a) – 8 (b) 8 (c) 1 (d) 7

$\vec{M_o} = \vec{OA} \times \vec{F}$

$= (2,1) \times (2,-3)$

$= (-6-2)\hat{k} = -8\hat{k}$

If force $\overrightarrow{F} = 2\,\hat{i} + 5\,\hat{j}$ acts at the point A = (– 3 , 1) , then the moment of $\overrightarrow{F}$ about the point N (2 , – 4) equals ....

(a) $15\,\hat{k}$ (b) $35\,\hat{k}$ (c) $-\hat{k}$ (d) $-35\,\hat{k}$

$$\overrightarrow{M_N} = \overrightarrow{NA} \times \overrightarrow{F}$$

$$= (-5, 5) \times (2, 5)$$

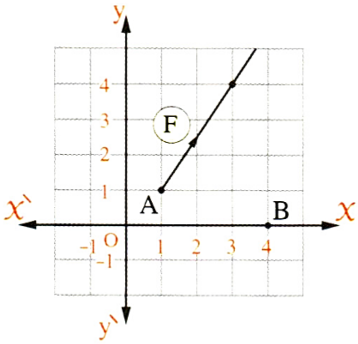
$$= (-25 - 10)\,\hat{k} = -35\,\hat{k}$$

In the opposite figure :

If $\overrightarrow{F} = 20\,\hat{i} + 30\,\hat{j}$ acts at the point A (1 , 1)

, then the moment of the force $\overrightarrow{F}$ about

the point B (4 , 0) = ... $\hat{k}$



(a) – 110 (b) 110

(c) 70 (d) – 90

$\overrightarrow{M_B} = \overrightarrow{BA} \times \overrightarrow{F}$

$= (-3, 1) \times (20, 30)$

$= (-90 - 20)\,\hat{k} = -110\,\hat{k}$

If $\vec{F} = 7\hat{j}$ acts at the point A $(-3, 0)$, then the moment of the force $\vec{F}$ about the point B $(1, -2)$ is ……………

(a) $-28\hat{k}$ (b) $28\hat{k}$ (c) $14\hat{k}$ (d) $-14\hat{k}$

$$\overrightarrow{M_B} = \overrightarrow{BA} \times \vec{F}$$

$$= (-4, 2) \times (0, 7)$$

$$= -28\hat{k}$$

If $\vec{F} = 6\,\hat{i} - 8\,\hat{j}$ acts at the point A (3 , – 2) , then the length of the perpendicular drawn from the point B (2 , 4) to the line of action of the force $\vec{F}$ = …………… length unit.

(a) 5.4 (b) 2.8 (c) 28 (d) 4.4

$$\vec{M}_B = \vec{BA} \times \vec{F}$$

$$= (1, -6) \times (6, -8)$$

$$= (-8 + 36)\,k = 28\,\hat{k}$$

$$L = \frac{\|\vec{M}_B\|}{\|\vec{F}\|} = \frac{28}{\sqrt{(6)^2 + (-8)^2}} = 2.8 \text{ L.u.}$$

If the line of action of force $\overrightarrow{F} \parallel \overline{AB}$, $\overrightarrow{M}_A = 12\,\hat{k}$, then $\overrightarrow{M}_B =$ …………… $\hat{k}$

(a) 12 (b) – 12 (c) 6 (d) 24

If the sum of the moments of some forces about A = the sum of the moments of these forces about B , then the line of action of their resultant ....

(a) is perpendicular to $\overline{AB}$

(b) is parallel to $\overline{AB}$

(c) bisects $\overline{AB}$

(d) coincide with $\overline{AB}$

$$\vec{M}_A = \vec{M}_B \Rightarrow \therefore \vec{R} \parallel \overline{AB}$$

If the sum of the moments of force $\vec{F}$ about two points A , B vanished , then the line of action of $\vec{F}$ ....

(a) is parallel to $\overline{AB}$

(b) is perpendicular to $\overline{AB}$

(c) passes through A or B

(d) bisects $\overline{AB}$

$$\vec{M}_A + \vec{M}_B = \vec{0}$$

$$\therefore \vec{M}_A = -\vec{M}_B$$

If $\vec{F} \neq \vec{O}$, then all the following are true except ....

(a) if the line of action of $\vec{F} \,//\, \overleftrightarrow{AB}$, then $\overrightarrow{M}_A - \overrightarrow{M}_B = \vec{O}$ $\overrightarrow{M}_A = \overrightarrow{M}_B$ ✓

(b) if the line of action of $\vec{F}$ bisects $\overline{AB}$, then $\overrightarrow{M}_A + \overrightarrow{M}_B = \vec{O}$ $\overrightarrow{M}_A = -\overrightarrow{M}_B$ ✓

(c) if $A \in$ the line of action of $\vec{F}$, then $\overrightarrow{M}_A \neq \vec{0}$

(d) if the line of action of $\vec{F}$ acts along $\overleftrightarrow{AB}$, then $\overrightarrow{M}_A = \overrightarrow{M}_B = \vec{O}$ ✓

If the force $\vec{F} = (\ell , m)$ acts at the point A (4 , 8) and the moment of $\vec{F}$ about B (3 , 9) equals $40\,\hat{k}$, then $\ell + m =$ ……………

(a) 40 (b) 20 (c) 10 (d) 80

$\vec{M}_B = \vec{BA} \times \vec{F} = 40\hat{k}$

$(1, -1) \times (\ell, m) = 40\hat{k}$

$(m + \ell)\hat{k} = 40\hat{k}$

$\therefore m + \ell = 40$

If $\vec{F} = 5\hat{i} + 12\hat{j}$ and its line of action has the equation $-12x + 5y =$ zero , then the moment of the force $\vec{F}$ about B $(-3, 1)$ equals .... $\hat{k}$

(a) zero (b) -11 (c) 31 (d) 41

let $A \in$ Line of action

Put $\boxed{x=0}$ $\therefore \boxed{y=0}$

$\therefore A = (0, 0)$

$\overrightarrow{M_B} = \overrightarrow{BA} \times \vec{F} = (3, -1) \times (5, 12)$

$= (36 + 6)k = 41\hat{k}$

If $\vec{F} = 5\hat{i} + 4\hat{j}$ and the two points A , B lie in the same plane as $\vec{F}$ where A (2 , 3) and $M_A = M_B$, then the equation of the straight line $\overleftrightarrow{AB}$ is ……………

(a) $4x - 5y + 7 = 0$ $m = \frac{-4}{-5} = \frac{4}{5}$

(b) $5x - 4y + 7 = 0$ $m = \frac{-5}{-4} = \frac{5}{4}$

(c) $4x - 5y = 0$ $m = \frac{-4}{-5} = \frac{4}{5}$

(d) $5x + 4y + 7 = 0$ $m = -\frac{5}{4}$

$\because M_A = M_B \Rightarrow \therefore \overleftrightarrow{AB} \parallel \vec{F}$

$\therefore$ Slope of $\overleftrightarrow{AB}$ = Slope of $\vec{F} = \frac{4}{5}$

$\because A \in L$ $\therefore A(2,3)$ Satisfies its equation

$4(2) - 5(3) + 7 = 0$

If the moment of the force $\vec{F} = 4\,\hat{i} + 6\,\hat{j}$ about the origin equals $80\,\hat{k}$, then the equation of the line of action of $\vec{F}$ is ....

(a) $2x + 3y = 40$ $m = \frac{-2}{3}$ ✗

(b) $4x + 3y = 10$ $m = -\frac{4}{3}$ ✗

(c) $3x - 2y = 40$ $m = \frac{-3}{-2} = \frac{3}{2}$

(d) $3x - 2y = 80$ $m = \frac{-3}{-2} = \frac{3}{2}$

$\vec{F} = 4\hat{i} + 6\hat{j} \Rightarrow m = \frac{6}{4} = \frac{3}{2}$

$\vec{M}_o = \vec{OA} \times \vec{F} = 80\hat{k}$

$(x, y) \times (4, 6) = 80$

$6x - 4y = 80$

$\boxed{3x - 2y = 40}$

The moment of force $\vec{F}$ about the point A $(3 , 5)$ is $6\hat{k}$ and its moment about the point B$(1 , -1)$ is $-6\hat{k}$, then its moment about the point .... $= \vec{0}$

(a) $(-1 , -3)$ (b) $(2 , 2)$ (c) $(2 , 6)$ (d) $(1 , 3)$

$\because \overrightarrow{M_A} = -\overrightarrow{M_B}$ $\therefore \vec{F}$ bisects $\overline{AB}$

$\therefore \vec{F}$ passes through the midpoint of $\overline{AB}$ which is C

$\therefore C = \dfrac{A+B}{2} = \dfrac{(3,5)+(1,-1)}{2} = (2,2)$

If the line of action of force $\vec{F}$, where $\vec{F} = \hat{i} + \hat{j}$, bisects $\overline{AB}$ where A (3 , – 1) and D (1 , 4) is the midpoint of $\overline{AB}$, then $\vec{M}_B$ = …………… $\hat{k}$

(a) – 7 (b) 7 (c) 3 (d) – 14

$\vec{M}_A = \vec{AD} \times \vec{F}$

$= (-2, 5) \times (1, 1)$

$= (-2 - 5)\hat{k} = -7\hat{k}$

$\therefore \vec{M}_B = 7\hat{k}$

If $\vec{F} = 3\,i - 2\,j$, A $(-1\ ,2)$, the moment of $\vec{F}$ about A is $\vec{M}_A = 9\,\vec{k}$, the moment of $\vec{F}$ about B is $\vec{M}_B = 9\,\vec{k}$, then the coordinates of the point B can be represented by one of the following ordered pairs except ……………

(a) $(5\ ,-2)$ (b) $(2\ ,0)$ (c) $(-8\ ,4)$ (d) $(8\ ,-4)$

$\because \vec{M}_A = \vec{M}_B$

$\therefore \vec{AB} \parallel \vec{F}$

$\therefore$ Slope of $\vec{AB}$ = Slope of $\vec{F}$

$$\frac{y-2}{x+1} = \frac{-2}{3}$$

$$3y - 6 = -2x - 2 \Rightarrow \boxed{2x + 3y = 4}$$

then try each choice

(Trial 2021) Force $\vec{F} = 3\hat{i} + 2\hat{j}$ acts at a point. The moment of $\vec{F}$ about origin is $15\hat{k}$, then intersection point of the line of action of $\vec{F}$ with the y-axis is ....

(a) $(0, -5)$ (b) $(0, 15)$ (c) $(0, 5)$ (d) $(0, -15)$

let $A \in$ y-axis $\Longrightarrow \therefore A = (0, b)$

$\overline{M}_o = \overrightarrow{OA} \times \vec{F} = 15\hat{k}$

$(0, b) \times (3, 2) = 15\hat{k}$

$-3b\hat{k} = 15\hat{k}$

$b = -5 \quad \therefore A = (0, -5)$

If the force $\vec{F} = (m , 7)$ acts at the point A $(1 , m)$ and its moment vector about B $(0 , 1)$ equals $5\hat{k}$, then $m \in$ ……………

(a) $\{-2 , 1\}$ (b) $\{-2 , -1\}$ (c) $\{2 , -1\}$ (d) $\{1 , 2\}$

$$\overrightarrow{M_B} = \overrightarrow{BA} \times \vec{F}$$

$$= (1 , m-1) \times (m , 7) = 5\hat{k}$$

$$(7 - m^2 + m)\hat{k} = 5\hat{k}$$

$$m^2 - m - 2 = 0$$

$m = 2$ $\underline{\text{or}}$ $m = -1$

Force $\overrightarrow{F} = 3\hat{i} + 4\hat{j}$ acts at the point A (2 , 9) and the point B (3 , 7) , then tangent of the angle between $\overrightarrow{BA}$, $\overrightarrow{F}$ = ...

(a) 2 (b) 4 (c) $\frac{2}{5}$ (d) $\frac{2}{\sqrt{5}}$

$\boxed{\text{From Solid}}$

$\sin\theta = \frac{\|\overrightarrow{A} \times \overrightarrow{B}\|}{\|\overrightarrow{A}\| \, \|\overrightarrow{B}\|}$, $\cos\theta = \frac{\overrightarrow{A} \cdot \overrightarrow{B}}{\|\overrightarrow{A}\| \, \|\overrightarrow{B}\|}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\|\overrightarrow{A} \times \overrightarrow{B}\|}{\overrightarrow{A} \cdot \overrightarrow{B}}$

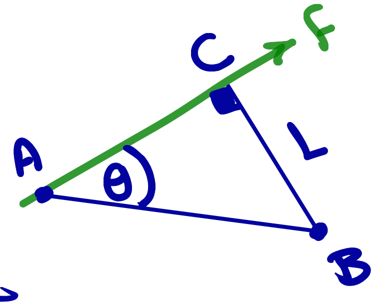
$\overrightarrow{BA} = (2,9) - (3,7) = (-1,2)$

$\overrightarrow{F} = (3,4)$

$\tan\theta = \frac{\|\overrightarrow{BA} \times \overrightarrow{F}\|}{\overrightarrow{BA} \cdot \overrightarrow{F}} = \frac{\|(-1,2) \times (3,4)\|}{(-1,2) \cdot (3,4)}$

$= \frac{\|-10\hat{k}\|}{-3+8} = \frac{10}{5} = 2$

Another sol.



$\overrightarrow{M_B} = \overrightarrow{BA} \times \overrightarrow{F} = (-1,2) \times (3,4)$

$= (-4-6)\hat{k} = -10\hat{k}$

$\|\overrightarrow{F}\| = \sqrt{(3)^2 + (4)^2} = 5 \Rightarrow L = \frac{\|\overrightarrow{M_B}\|}{\|F\|} = \frac{10}{5} = 2$

$\|\overrightarrow{AB}\| = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$ $\therefore AC = \sqrt{5-4} = 1$

$\therefore \tan\theta = \frac{BC}{AC} = \frac{2}{1} = 2$

If $\overrightarrow{F} = 2\hat{i} + 3\hat{j}$ acts at point C and $\overrightarrow{AB} = 4\hat{i} + 6\hat{j}$ and $\overrightarrow{M_A} = (m^2 + 4)\hat{k}$, $\overrightarrow{M_B} = (4\,m)\hat{k}$, then m = ...

(a) 3 (b) 2 (c) 4 (d) 1

Slope of $\overrightarrow{F} = \frac{3}{2}$, Slope of $\overrightarrow{AB} = \frac{6}{4} = \frac{3}{2}$

$\because m_1 = m_2 = \frac{3}{2}$

$\therefore \overrightarrow{F} \parallel \overrightarrow{AB}$ $\therefore \overrightarrow{M_A} = \overrightarrow{M_B}$

$\therefore m^2 + 4 = 4m$

$m^2 - 4m + 4 = 0$

$\therefore m = 2$

The force $\overrightarrow{F}$ acts at point A , the points A , B , C lies on the same plane as $\overrightarrow{F}$ and $\overrightarrow{M_B} = -12\,\hat{k}$, $\overrightarrow{BC} \times \overrightarrow{F} = 23\,\hat{k}$, then $\overrightarrow{M_C} =$ ....

(a) $-11\,\hat{k}$ (b) $11\,\hat{k}$ (c) $-35\,\hat{k}$ (d) $35\,\hat{k}$

$$\overrightarrow{M_c} = \overrightarrow{CA} \times \overrightarrow{F}$$

$$= (\overrightarrow{CB} + \overrightarrow{BA}) \times \overrightarrow{F}$$

$$= \overrightarrow{CB} \times \overrightarrow{F} + \overrightarrow{BA} \times \overrightarrow{F}$$

$$= -\overrightarrow{BC} \times \overrightarrow{F} + \overrightarrow{M_B}$$

$$= -23\hat{k} - 12\hat{k} = -35\hat{k}$$

If the force $\vec{F} = \left(10 , \frac{\pi}{3}\right)$ acts at the point A $\left(\sqrt{3} , 2\right)$, then the moment of the force $\vec{F}$ about the origin "O" equals ....

(a) $-5\,\hat{k}$ (b) $5\,\hat{k}$ (c) $5\sqrt{3}\,\hat{k}$ (d) $-25\,\hat{k}$

$$\vec{F} = \left(10\cos\frac{\pi}{3} , 10\sin\frac{\pi}{3}\right) = (5, 5\sqrt{3})$$

$$\vec{M_o} = \vec{OA} \times \vec{F}$$

$$= (\sqrt{3}, 2) \times (5, 5\sqrt{3})$$

$$= (15 - 10)\hat{k} = 5\hat{k}$$

Exercise 2

The moment of a force about a point in 2D-coordinate system

First : Moment by using vector product

Answer the following questions

1. If $\overrightarrow{F} = \hat{i} - 2\hat{j}$ act at the point A (2 , 3)

Find : (1) Moment of $\overrightarrow{F}$ about the point B (2 , 1)

(2) Perpendicular length from the point B to the line of action of $\overrightarrow{F}$

(1) $\overrightarrow{M_B} = \overrightarrow{BA} \times \overrightarrow{F}$

$= (0,2) \times (1,-2) = -2\hat{k}$

(2) $L = \frac{\|\overrightarrow{M_B}\|}{\|\overrightarrow{F}\|} = \frac{\|-2\hat{k}\|}{\sqrt{1+4}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ l.u.

(2) If $\vec{F} = 3\hat{i} - 4\hat{j}$ acts at the point A = (0 , 2) and if B = (3 , − 2) , C = (2 , 3) , D = (− 2 , 1) , E = (5 , − 1) Prove using moments that the action line of $\vec{F}$

(1) Passes through the point B (2) Bisects $\overline{CD}$ (3) Parallel to $\overleftrightarrow{CE}$

(1) $\vec{M}_B = \vec{BA} \times \vec{F} = (-3, 4) \times (3, -4)$

$= (12 - 12)\hat{k} = 0\hat{k}$

∴ line of $\vec{F}$ passes through B

(2) $\vec{M}_C = \vec{CA} \times \vec{F} = (-2, -1) \times (3, -4)$

$= (8 + 3)\hat{k} = 11\hat{k}$

$\vec{M}_D = \vec{DA} \times \vec{F} = (2, 1) \times (3, -4)$

$= (-8 - 3)\hat{k} = -11\hat{k}$

∵ $\vec{M}_C = -\vec{M}_D$

∴ line of $\vec{F}$ bisects $\overline{CD}$

(3) $\vec{M}_C = 11\hat{k}$

$\vec{M}_E = \vec{EA} \times \vec{F} = (-5, 3) \times (3, -4)$

$= (20 - 9)\hat{k} = 11\hat{k}$

∵ $\vec{M}_C = \vec{M}_E$

∴ line of $\vec{F}$ // $\overline{CE}$

(3) The forces $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{F_2} = 5\hat{i} + \hat{j}$, $\overrightarrow{F_3} = -4\hat{i} + 7\hat{j}$ act at the points A (1 , 1) , B (– 2 , 2) , C (3 , 1) respectively. Find the moment vector of the resultant about the origin (0 , 0) « $8\hat{k}$ »

$$\overrightarrow{M_1} = \overrightarrow{OA} \times \overrightarrow{F_1} = (1,1) \times (2,-3) = -5\hat{k}$$

$$\overrightarrow{M_2} = \overrightarrow{OB} \times \overrightarrow{F_2} = (-2,2) \times (5,1) = -12\hat{k}$$

$$\overrightarrow{M_3} = \overrightarrow{OC} \times \overrightarrow{F_3} = (3,1) \times (-4,7) = 25\hat{k}$$

$$\overrightarrow{M_1} + \overrightarrow{M_2} + \overrightarrow{M_3} = (-5 - 12 + 25)\hat{k}$$

$$= 8\hat{k}$$

4 The forces $\overrightarrow{F_1} = 2\hat{i} - \hat{j}$, $\overrightarrow{F_2} = 5\hat{i} + 2\hat{j}$, $\overrightarrow{F_3} = -3\hat{i} + 2\hat{j}$ act at the point , A (1 , 1) Prove using the moments that the line of action of the resultant is parallel to the straight line passing through the two points (2 , 1) and (6 , 4)
C D

$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = (2,-1) + (5,2) + (-3,2)$

$= (4,3)$

$\overrightarrow{M_C} = \overrightarrow{CA} \times \overrightarrow{R} = (-1,0) \times (4,3) = -3\hat{k}$

$\overrightarrow{M_D} = \overrightarrow{DA} \times \overrightarrow{R} = (-5,-3) \times (4,3) = -3\hat{k}$

$\because \overrightarrow{M_C} = \overrightarrow{M_D}$

$\therefore \overline{CD} \parallel$ line of action of R

5 The two forces $\overrightarrow{F_1} = m\,\hat{i} + 2\,\hat{j}$, $\overrightarrow{F_2} = L\,\hat{i} - \hat{j}$ act at the points $A_1 = (1\ ,1)$, $A_2 = (-1\ ,-2)$ respectively. Find the values of the constants m , L if the sum of their moments about each of the origin and about the point B = (2 , 3) vanishes. «$\frac{13}{9}$, $-\frac{7}{9}$»

$$\overrightarrow{M_o} = \overrightarrow{OA_1} \times \overrightarrow{F_1} + \overrightarrow{OA_2} \times \overrightarrow{F_2} = \overrightarrow{0}$$

$$= (1,1) \times (m,2) + (-1,-2) \times (L,-1) = \overrightarrow{0}$$

$$(2-m)\hat{k} + (1+2L)\hat{k} = \overrightarrow{0}$$

$$2 - m + 1 + 2L = 0$$

$$\boxed{2L - m = -3} \longrightarrow \text{I}$$

$$\overrightarrow{M_B} = \overrightarrow{BA_1} \times \overrightarrow{F_1} + \overrightarrow{BA_2} \times \overrightarrow{F_2} = \overrightarrow{0}$$

$$= (-1,-2) \times (m,2) + (-3,-5) \times (L,-1) - \overrightarrow{0}$$

$$(-2+2m)\hat{k} + (3+5L)\hat{k} = 0$$

$$5L + 2m + 1 = 0$$

$$\boxed{5L + 2m = -1} \longrightarrow \text{II}$$

$$L = -\frac{7}{9} \quad , \quad m = \frac{13}{9}$$