On friction – equilibrium of body placed on a horizontal rough plane

1

$\because$ The friction is limiting

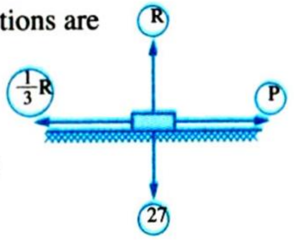
$\therefore$ The two equilibrium equations are

$P = \frac{1}{3} R \qquad (1)$

$R = 27 \qquad (2)$

Substituting from (2) in (1) :

$\therefore P = \frac{1}{3} \times 27 = 9$ kg. wt.

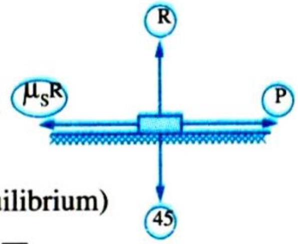


2

The horizontal force which makes the body about to begin motion is $P = \mu_s R$

$\because R = 45$ (the body is in equilibrium)

$\therefore P = \frac{\sqrt{3}}{3} \times 45 \quad \therefore P = 15\sqrt{3}$ kg. wt.



The resultant reaction

$\grave{R} = R\sqrt{1 + \mu_s^2} = 45\sqrt{1 + \frac{1}{3}} = 45 \times \frac{2}{\sqrt{3}}$

$\therefore \grave{R} = \frac{90}{\sqrt{3}} = 30\sqrt{3}$ kg. wt.

$\tan \lambda = \mu_s = \frac{\sqrt{3}}{3}$

$\therefore$ The measure of the angle of the friction $\lambda = 30°$

$\therefore$ The resultant reaction makes with the vertical an angle of measure 30°

3

$\because$ The suspended body is in equilibrium

$\therefore T = 1.5$ kg.wt.

$\because$ The body on the table is in equilibrium

$\therefore F = T = 1.5$ kg. wt.

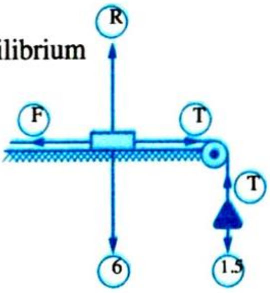
, $R = 6$

$\because \mu_s R = 6 \times \frac{1}{3} = 2$ kg. wt.

$\therefore F < \mu_s R$

$\therefore$ The friction is not limiting.

$\therefore$ The body is not about to begin motion.



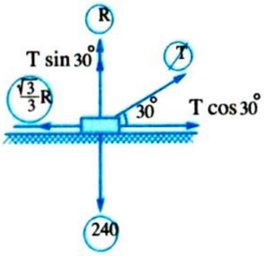
4

$\because$ The body is about to begin motion.

$\therefore T \cos 30° = \frac{\sqrt{3}}{3} R$

$\therefore \frac{\sqrt{3}}{2} T \times \frac{3}{\sqrt{3}} = R$

$\therefore R = \frac{3}{2} T \qquad (1)$



$R + T \sin 30° = 240 \qquad \therefore R + \frac{1}{2} T = 240$

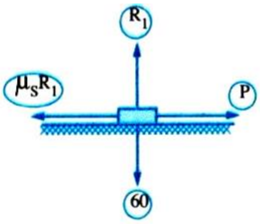
Substituting in (1) :

$\therefore \left(\frac{3}{2} + \frac{1}{2}\right) T = 240 \qquad \therefore 2T = 240$

$\therefore T = 120$ kg. wt.

5

- **In the case of the horizontal force the coefficient of friction :**



$\mu_s = \tan \lambda = \tan 30° = \frac{1}{\sqrt{3}}$

$\because$ The body is about to begin motion

$\therefore P = \mu_s R_1 , R_1 = 60°$

$\therefore P = \frac{1}{\sqrt{3}} \times 60 = 20\sqrt{3}$ gm. wt.

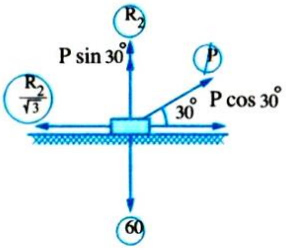
- **In the case of inclined force :**

$\because$ The body is about to begin motion.

$\therefore P \cos 30° = \frac{1}{\sqrt{3}} R_2$

$\therefore \frac{\sqrt{3}}{2} P = \frac{1}{\sqrt{3}} R_2$

$\therefore R_2 = \frac{3}{2} P \qquad (1)$

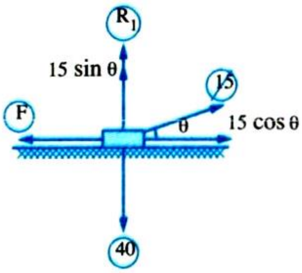


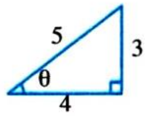
$\therefore R_2 + P \sin 30° = 60$

Substituting in (1) : $\therefore \frac{3}{2} P + \frac{1}{2} P = 60$

$\therefore 2P = 60 \qquad \therefore P = 30$ gm.wt.

6

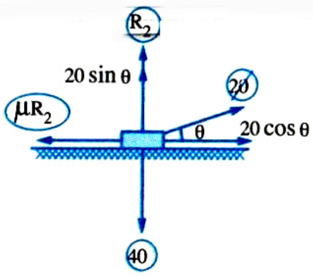




$\because$ The body is in equilibrium

$\therefore F = 15 \cos\theta = 15 \times \frac{4}{5}$

$= 12$ newton.



After increasing the force

$\because$ The body is about to begin motion.

$\therefore 20 \cos\theta = \mu_s R_2$

$\therefore 20 \times \frac{4}{5} = \mu_s R_2 \qquad \therefore \mu_s R_2 = 16 \qquad (1)$

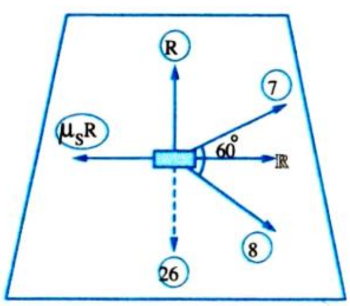
$20 \sin\theta + R_2 = 40 \qquad \therefore 20 \times \frac{3}{5} + R_2 = 40$

$R_2 = 28$

Substituting in (1) :

$\therefore \mu_s \times 28 = 16 \qquad \therefore \mu_s = \frac{16}{28} = \frac{4}{7}$

7



$\mathbb{R} = \sqrt{64 + 49 + 2 \times 8 \times 7 \times \frac{1}{2}} = 13$ gm.wt.

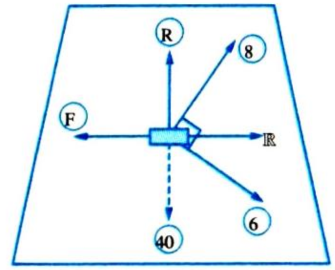
$\because$ The body is about to begin motion

$\therefore 13 = \mu_s R$, $R = 26 \qquad \therefore 13 = 26 \mu_s$

$\therefore \mu_s = \frac{1}{2} \qquad \tan\lambda = \frac{1}{2}$

$\therefore$ The measure of the angle of friction $(\lambda) = 26° 34'$

8



$\mathbb{R} = \sqrt{64 + 36} = 10$

$\because$ The body is in equilibrium.

$\therefore F = 10$

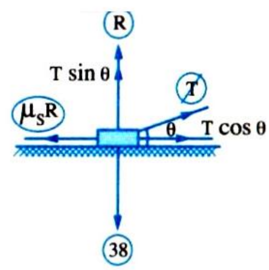
$R = 40$

$\because F \le \mu_s R$ (the body is in equilibrium)

$\therefore 10 \le 40 \mu_s \qquad \therefore \mu_s \ge \frac{1}{4}$

$\therefore$ The static coefficient of friction should not be less than $\frac{1}{4}$

9

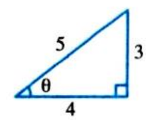


- **On the inclined plane :**

$\because$ The body is about to slide under the effect of its weight only

$\therefore \mu_s = \tan\theta$

$\therefore \mu_s = \frac{1}{4}$



- **On the horizontal plane :**

$\because$ The body is about to move

$\therefore$ The equations of equilibrium are

$T \cos\theta = \mu_s R \qquad \therefore \frac{4}{5} T = \frac{1}{4} R$

$\therefore R = \frac{16}{5} T \qquad (1)$

$R + T \sin\theta = 38$

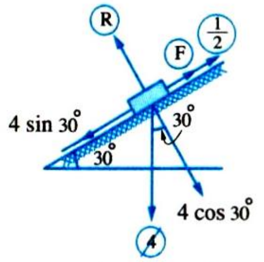
Substituting from (1) :

$\therefore \frac{16}{5} T + \frac{3}{5} T = 38 \qquad \therefore \frac{19}{5} T = 38$

$\therefore T = 38 \times \frac{5}{19} = 10$ kg.wt.

$R = \frac{16}{5} \times 10 = 32$ kg.wt.

(10)



$\because W \sin\theta = 4 \times \sin 30° = 2$

$\therefore W \sin\theta > P$

$\therefore \vec{F}$ acts in the direction of the line of the greatest slope upwards.

$\because$ The body is in equilibrium. $\therefore F + \frac{1}{2} = 4 \sin 30°$

$\therefore F + \frac{1}{2} = 2$ $\qquad \therefore F = 1.5$ newton.

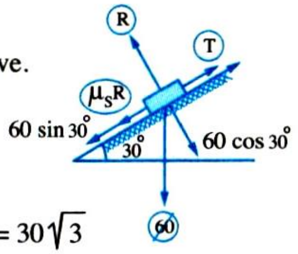
$R = 4 \cos 30° = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

$\therefore \mu_s R = \frac{\sqrt{3}}{4} \times 2\sqrt{3} = \frac{3}{2} = 1.5$ newton.

$\therefore F = \mu_s R$

$\therefore$ The body is about to start motion.

(11)



$\because$ The body is about to move.

$\therefore T = \mu_s R + 60 \sin 30°$

$\therefore T = \frac{1}{\sqrt{3}} \times R + 30 \quad (1)$

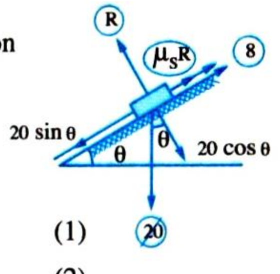
$R = 60 \cos 30° = 60 \times \frac{\sqrt{3}}{2} = 30\sqrt{3}$

Substituting in (1) :

$\therefore T = \frac{1}{\sqrt{3}} \times 30\sqrt{3} + 30$

$\therefore T = 30 + 30 = 60$ kg.wt.

(12)



$\because$ The body is about to motion

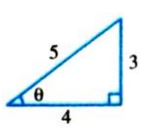
$\therefore P + \mu_s R = W \sin\theta$

$\therefore 8 + \mu_s R = 20 \times \frac{3}{5}$

$\therefore 8 + \mu_s R = 12$

$\therefore \mu_s R = 4 \quad (1)$

$R = 20 \cos\theta = \frac{20}{1} \times \frac{4}{5} = 16 \quad (2)$



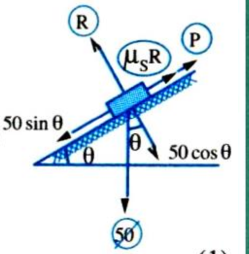
From (1) and (2) : $\therefore \mu_s \times 16 = 4$

$\therefore \mu_s = \frac{1}{4}$ $\qquad \therefore \tan\lambda = \frac{1}{4}$

$\therefore$ The measure of the angle of friction

$\lambda \simeq 14° \, 2'$

(13)



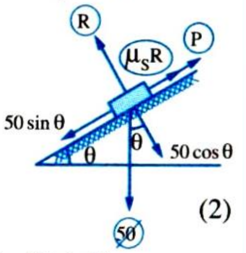
When the body is about to move downwards (the least force)

$\therefore P + \mu_s R = 50 \sin\theta$, $R = 50 \cos\theta$

$\therefore 10 + \mu_s \times 50 \cos\theta = 50 \sin\theta$

$\therefore \mu_s = \frac{50 \sin\theta - 10}{50 \cos\theta} \quad (1)$

and when the body is about to move upwards. (The greatest force)



$\therefore P = \mu_s R + 50 \sin\theta$

, $R = 50 \cos\theta$

$40 = \mu_s \times 50 \cos\theta + 50 \sin\theta$

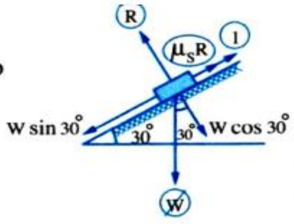
$\therefore \mu_s = \frac{40 - 50 \sin\theta}{50 \cos\theta} \quad (2)$

From (1) , (2) : $\frac{50 \sin\theta - 10}{50 \cos\theta} = \frac{40 - 50 \sin\theta}{50 \cos\theta}$

$\therefore 100 \sin\theta = 50$ $\qquad \therefore \sin\theta = \frac{1}{2}$

$\therefore \theta = 30°$ $\qquad \therefore \mu_s = \frac{\sqrt{3}}{5}$

(14)



• When the body is about to move downwards.

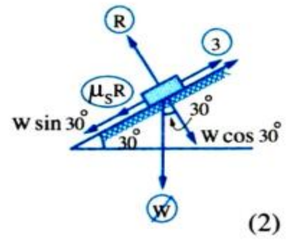
$\therefore 1 + \mu_s R = W \sin 30°$

$R = W \cos 30°$

$\therefore 1 + \mu_s W \cos 30° = W \sin 30°$

$\therefore 1 + \frac{\sqrt{3}}{2} \mu_s W = \frac{1}{2} W$ $\qquad \therefore \mu_s = \frac{\frac{1}{2} W - 1}{\frac{\sqrt{3}}{2} W} \quad (1)$

• When the body is about to move upwards.



$\therefore 3 = \mu_s R + W \sin 30°$

$\therefore 3 = \mu_s \times \frac{\sqrt{3}}{2} W + \frac{1}{2} W$

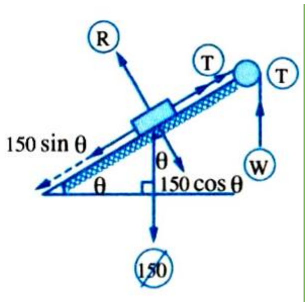
$\therefore \mu_s = \frac{3 - \frac{1}{2} W}{\frac{\sqrt{3}}{2} W} \quad (2)$

From (1) and (2) : $\therefore \frac{\frac{1}{2} W - 1}{\frac{\sqrt{3}}{2} W} = \frac{3 - \frac{1}{2} W}{\frac{\sqrt{3}}{2} W}$

$\therefore \frac{1}{2} W - 1 = 3 - \frac{1}{2} W$ $\qquad \therefore W = 4$ kg.wt.

$\mu_s = \frac{3-2}{2\sqrt{3}} = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $\qquad \therefore \mu_s = \frac{\sqrt{3}}{6}$

15



- When $W = 125\sqrt{2}$

$\therefore T = 125\sqrt{2}$

$\therefore$ The body is about to move upwards.

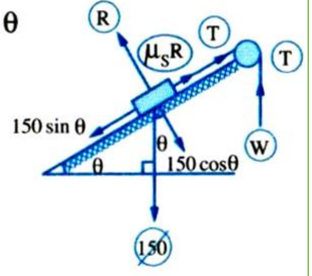
$\therefore R = 150 \cos\theta$

$125\sqrt{2} = \mu_s R + 150 \sin\theta$

$125\sqrt{2} = \mu_s \times 150 \cos\theta + 150 \sin\theta$

$\therefore 5\sqrt{2} = 6\mu_s \cos\theta + 6 \sin\theta$

$\therefore \mu_s = \dfrac{5\sqrt{2} - 6\sin\theta}{6\cos\theta}$ (1)



- When $W = 25\sqrt{2}$

$\therefore T = 25\sqrt{2}$

$\therefore$ The body is about to move downwards.

$\therefore 25\sqrt{2} + \mu_s R = 150 \sin\theta$

$\therefore 25\sqrt{2} + \mu_s \times 150 \cos\theta = 150 \sin\theta$

$\therefore \sqrt{2} + 6\mu_s \cos\theta = 6 \sin\theta$

$\therefore \mu_s = \dfrac{6\sin\theta - \sqrt{2}}{6\cos\theta}$ (2)

From (1) and (2) : $\therefore 6\sin\theta - \sqrt{2} = 5\sqrt{2} - 6\sin\theta$

$\therefore 12 \sin\theta = 6\sqrt{2}$ $\quad \therefore \sin\theta = \dfrac{1}{\sqrt{2}}$

$\therefore \theta = 45°$ $\quad \therefore \mu_s = \dfrac{3\sqrt{2} - \sqrt{2}}{3\sqrt{2}} = \dfrac{2}{3}$