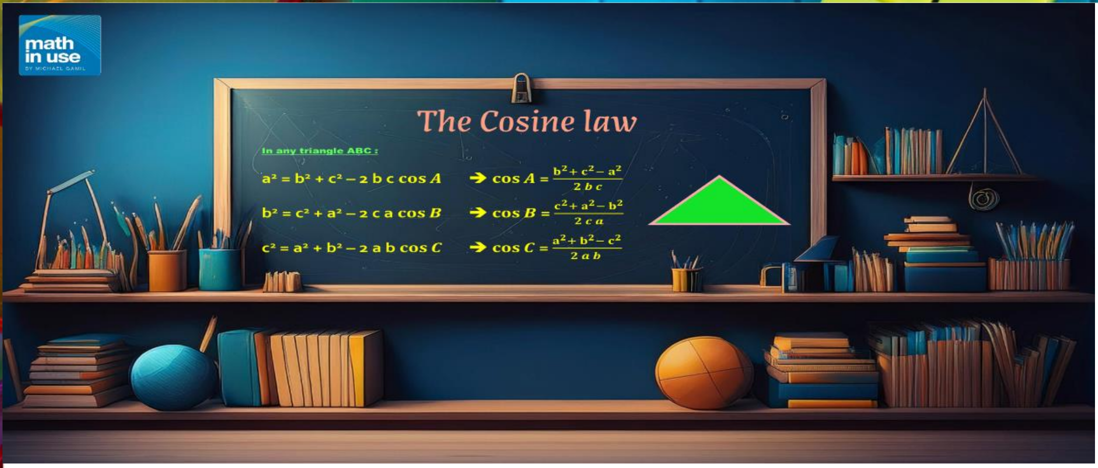


Exercise 2

The Cosine rule

Choose the correct answer

In Δ XYZ , the expression $\frac{x^2+y^2-z^2}{2xy}$ equals ....

(a) cos X (b) cos Y (c) cos Z (d) sin Z

Mr. Michael Gamil 0122 73 75 987

Choose the correct answer

In Δ XYZ , $y^2 + z^2 - x^2 = 2\,y\,z \times$ ………….

(a) cos X (b) sin Z (c) cos Z (d) sin X

$$\frac{y^2+z^2-x^2}{2yz} = \cos x$$

$$y^2+z^2-x^2 = 2yz \times \cos x$$

Choose the correct answer

In Δ ABC , cos (A + B) = ...

(a) cos C (b) – cos C (c) sin C (d) – sin C

$A + B + C = 180$

$A + B = 180 - C$

$\cos(A+B) = \cos(180 - C)$

$\cos(A+B) = -\cos C$

Choose the correct answer

$-\cos C + \cos C$

If ABCD is a cyclic quadrilateral , then $\cancel{\cos} A + \cos C =$ ....

(a) 1 (b) zero. (c) $\frac{1}{2}$ (d) -1

$A + C = 180$

$A = 180 - C$

$\cos A = \cos(180 - C)$

$\boxed{\cos A = -\cos C}$

Choose the correct answer

In Δ XYZ , $2\,x\,y\cos(X+Y) = \ldots\ldots\ldots\ldots$

(a) $x^2 + y^2 - z^2$ (b) $y^2 + z^2 - x^2$ (c) $x^2 - z^2 - y^2$ (d) $z^2 - x^2 - y^2$

$$2xy \times -\cos Z$$

$$= -\cancel{2xy} \times \frac{x^2+y^2-z^2}{\cancel{2xy}}$$

$$= -\left[x^2+y^2-z^2\right] = -x^2 - y^2 + z^2$$

$$= z^2 - x^2 - y^2$$

Choose the correct answer

S'. A. S'

In Δ LMN , $\ell = 5$ cm. , m = 7 cm. , m ($\angle$ N) = 60°

, then n = ... cm. (to the nearest tenth)

(a) 6.2 (b) 5 (c) 4.3 (d) 3.5

$$n^2 = \ell^2 + m^2 - 2\ell m \cos N$$

$$n = \sqrt{(5)^2 + (7)^2 - 2(5)(7)\cos 60}$$

$$= \sqrt{39} \simeq 6.2$$

Choose the correct answer

$m(\angle Z) = \frac{2}{3} \times 180^\circ = 120^\circ$

In ΔXYZ , $x = 5$ cm. , $y = 3$ cm. , $m(\angle Z) = \frac{2}{3}\pi$, then $z =$ ....

(a) 7 (b) 8 (c) 9 (d) 4

$$Z^2 = x^2 + y^2 - 2xy\cos Z$$

$$Z = \sqrt{(5)^2 + (3)^2 - 2(5)(3)\cos 120} = 7$$

Choose the correct answer

In ΔABC , if $m(\angle A) + m(\angle B) = 120^\circ$, $a = 2$ cm. , $b = 3$ cm. , then $c =$ .... cm.

(a) 4 (b) 3 (c) $\sqrt{7}$ (d) $\sqrt{5}$

$m(\angle C) = 180^\circ - 120^\circ = 60^\circ$

$c^2 = a^2 + b^2 - 2ab \cos C$

$c = \sqrt{(2)^2 + (3)^2 - 2(2)(3)\cos 60} = \sqrt{7}$

Choose the correct answer

In ΔABC , $a = 9$ cm. , $b = 15$ cm. , $m(\angle C) = 106°$
, then its perimeter $\simeq$ .... cm.

(a) 44 (b) 24 (c) 34 (d) 28

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c = \sqrt{(9)^2 + (15)^2 - 2(9)(15)\cos 106} \simeq 20$$

$$P = a + b + c = 9 + 15 + 20 = 44$$

Choose the correct answer

In ΔABC , $b = 2$ cm. , $c = 2.5$ cm. , $\cos A = \frac{2}{5}$, then ΔABC is ……………

(a) a right angled triangle.

(b) an isosceles triangle.

(c) an equilateral triangle.

(d) a scalene.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a = \sqrt{(2)^2 + (2.5)^2 - 2(2)(2.5)\left(\frac{2}{5}\right)} = 2.5$$

Choose the correct answer

In Δ XYZ , if $x = y$, then cos X = ...

(a) $\frac{2y^2}{z}$ (b) $\frac{z}{2y}$ (c) $\frac{z}{4x}$ (d) $\frac{y}{2x}$

$$\cos X = \frac{\cancel{y^2} + z^2 \cancel{- x^2}}{2yz} = \frac{z^{\cancel{2}}}{2y\cancel{z}} = \frac{z}{2y}$$

Choose the correct answer

In Δ ABC , cos (A + B) = ...

(a) $\frac{a^2+b^2-c^2}{2ab}$ (b) $\frac{a^2+c^2-b^2}{2ab}$ (c) $\frac{b^2+c^2-a^2}{2bc}$ (d) $\frac{c^2-a^2-b^2}{2ab}$

$$\cos(A+B) = -\cos C$$

$$= -\left[\frac{a^2+b^2-c^2}{2ab}\right]$$

$$= \frac{c^2-a^2-b^2}{2ab}$$

math in use BY MICHAEL GAMIL

Choose the correct answer

The measure of the greatest angle in triangle the lengths of its sides are 3 cm. a , b 5 cm. , c 7 cm. equals ....°

(a) 110 (b) 150 (c) 100 (d) 120

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{(3)^2 + (5)^2 - (7)^2}{2(3)(5)} = -\frac{1}{2}$$

$$m(\angle C) = \cos^{-1}\left(-\frac{1}{2}\right) = 120°$$

Choose the correct answer

$(6)^2 \neq (4)^2 + (5)^2$
$36 \neq 41$

In Δ ABC , b = 4 cm. , a + c = 11 cm. , a – c = 1 cm. , then ……………

~~(a)~~ the triangle is an obtuse angled triangle.

~~(b)~~ the triangle is a right angled triangle.

(c) m (∠ B) = 2 m (∠ A)

(d) m (∠ A) = 2 m (∠ B)

$$a + c = 11$$
$$a - c = 1$$
$$2a = 12 \quad \boxed{\therefore a = 6}$$
$$\boxed{c = 5}$$

$a = 6$ $\quad b = 4$ $\quad c = 5$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{16 + 25 - 36}{2(4)(5)} = \frac{1}{8}$$

$$m(\angle A) = 82^\circ\ 49'$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{25 + 36 - 16}{2(5)(6)} = \frac{3}{4}$$

$$m(\angle B) = 41^\circ\ 24'$$

Choose the correct answer

In Δ ABC , c (a cos B + b cos A) = ……………

(a) $2c^2$ (b) c^2 (c) a^2 (d) b^2

$$c\left[a \cdot \frac{c^2+a^2-b^2}{2ca} + b \cdot \frac{b^2+c^2-a^2}{2bc}\right]$$

$$\cancel{c}a \cdot \frac{c^2+a^2-b^2}{2\cancel{c}a} + \cancel{b}c \cdot \frac{b^2+c^2-a^2}{2\cancel{b}c}$$

$$= \frac{c^2+a^2-b^2}{2} + \frac{b^2+c^2-a^2}{2}$$

$$= \frac{c^2+\cancel{a^2}-\cancel{b^2}+\cancel{b^2}+c^2-\cancel{a^2}}{2} = \frac{\cancel{2}c^2}{\cancel{2}} = c^2$$

Choose the correct answer

In Δ ABC , if $\frac{\sin A}{\sin B} = 2 \cos C$, then ....

(a) b = c (b) a = c (c) a = b (d) a = b = c

$$\frac{\sin A}{\sin B} = \frac{a}{b} = 2\cos c$$

$$a = 2b.\cos c$$

$$a = \cancel{2b} \cdot \frac{a^2+b^2-c^2}{\cancel{2}a\cancel{b}}$$

$$\frac{a}{1} = \frac{a^2+b^2-c^2}{a}$$

$$\cancel{a^2} = \cancel{a^2}+b^2-c^2$$

$$c^2 = b^2$$

$$c = b$$

Choose the correct answer

In Δ ABC , $a^2 + b^2 - c^2 + \sqrt{3}\ ab = 0$, then m (∠ C) = …………… °

(a) 30 (b) 150 (c) 60 (d) 120

$$\frac{a^2+b^2-c^2}{2ab} = -\frac{\sqrt{3}\ \cancel{ab}}{2\cancel{ab}}$$

$$\cos(\angle C) = -\frac{\sqrt{3}}{2}$$

$$m(\angle C) = 150^\circ$$

Choose the correct answer

In Δ ABC , if m ($\angle$ C) = 60° , $a^2 + b^2 - c^2 = k\,a\,b$, then k = ...

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) – 1

$$\frac{a^2+b^2-c^2}{2ab} = \frac{K a \cancel{b}}{2 \cancel{a b}}$$

$$\cos C = \frac{K}{2}$$

$$\cos 60 = \boxed{\frac{K}{2} = \frac{1}{2}} \Rightarrow K = 1$$

Choose the correct answer

In triangle ABC , $c^2 = (a + b)^2 - ab$, then m (∠ C) ....°

(a) 30 (b) 45 (c) 60 (d) 120

$$c^2 = a^2 + 2ab + b^2 - ab$$

$$c^2 = a^2 + b^2 + ab$$

$$\frac{a^2 + b^2 - c^2}{2ab} = \frac{-\cancel{ab}}{2\cancel{ab}}$$

$$\cos C = -\frac{1}{2}$$

$\therefore m(\angle C) = 120^\circ$

Choose the correct answer

In Δ ABC , 4 sin A = 3 sin B = 6 sin C , then m ($\angle$ C) = ...
(to the nearest degree)

(a) 89° (b) 29° (c) 57° (d) 82°

$$\frac{4\sin A}{12} = \frac{3\sin B}{12} = \frac{6\sin C}{12}$$

3, 4, 6 | 2
3, 2, 3 | 2
3, 1, 3 | 3
1, 1, 1

$$\frac{\sin A}{3} = \frac{\sin B}{4} = \frac{\sin C}{2}$$

$$a : b : c = 3 : 4 : 2$$

$a = 3m$ $b = 4m$ $c = 2m$

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{9m^2+16m^2-4m^2}{2(3m)(4m)} = \frac{7}{8}$$

$$m(\angle C) = 29°$$

Choose the correct answer

In Δ ABC , $\frac{1}{2}$ sin A = $\frac{1}{3}$ sin B = $\frac{1}{4}$ sin C , then cos C = ...

(a) $\frac{-2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{-1}{4}$ (d) $\frac{1}{4}$

$$\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{4}$$

$a : b : c = 2 : 3 : 4$

$a = 2m$, $b = 3m$, $c = 4m$

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{4m^2+9m^2-16m^2}{2(2m)(3m)}$$

$$= \frac{-3m^2}{12m^2} = -\frac{1}{4}$$

Choose the correct answer

If ABC is a triangle in which : 5 sin A sin B = 6 sin B sin C = 9 sin C sin A ,
then m (∠ C) ≃ ………….°

(a) 28 (b) 32 (c) 36 (d) 42

$$\frac{5\cancel{\sin A}\cancel{\sin B}}{\cancel{\sin A}\cancel{\sin B}\sin C} = \frac{6\cancel{\sin B}\cancel{\sin C}}{\sin A\cancel{\sin B}\cancel{\sin C}} = \frac{9\cancel{\sin C}\cancel{\sin A}}{\cancel{\sin A}\sin B\cancel{\sin C}}$$

$$\frac{5}{\sin C} = \frac{6}{\sin A} = \frac{9}{\sin B}$$

$$a : b : c = 6 : 9 : 5$$

$a = 6m$, $b = 9m$, $c = 5m$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{36m^2 + 81m^2 - 25m^2}{2(6m)(9m)}$$

$$\cos C = \frac{23}{27}$$

$$m(\angle C) = \cos^{-1}\left(\frac{23}{27}\right) = 31^\circ\ 35'\ 11''$$

Choose the correct answer

If ABC is a triangle in which : 6 a = 4 b = 3 c , then the measure of the smallest angle in the triangle ≃ ....

(a) 57° 28`    (b) 41° 12`   (c) 28° 57`    (d) 36° 52`

$$\frac{6a}{12} = \frac{4b}{12} = \frac{3c}{12} \Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{4}$$

$$a : b : c = 2 : 3 : 4$$

$a = 2m$ $b = 3m$ $c = 4m$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

$$= \frac{9m^2 + 16m^2 - 4m^2}{2(3m)(4m)} = 7/8$$

$$m(\angle A) = \cos^{-1}\left(\frac{7}{8}\right)$$

$$= 28^\circ\, 57`$$

Choose the correct answer

$b = 5m$
$c = 8m$

ABC is a triangle in which m (∠ A) = 60° , b : c = 5 : 8 and the area of the circumcircle of the triangle ABC is 147 π cm^2, then the perimeter of Δ ABC = ... cm.

(a) 21 (b) 34 (c) 54 (d) 60

$\cancel{\pi} r^2 = 147 \cancel{\pi}$

$r = \sqrt{147} = 7\sqrt{3}$ cm $\Longrightarrow$

$\frac{a}{2\sin A} = r$

$\frac{a}{2\sin 60} = 7\sqrt{3}$

$a = 2\sin 60 \times 7\sqrt{3}$

$\boxed{a = 21 \text{ cm}}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$(21)^2 = (5m)^2 + (8m)^2 - \cancel{2}(5m)(8m)\cancel{\cos 60}$

$441 = 25m^2 + 64m^2 - 40m^2$

$441 = 49m^2 \Longrightarrow m^2 = 9$

$\boxed{m = 3}$

$b = 5m = 5(3) = 15$

$c = 8m = 8(3) = 24$

P. of $\Delta ABC = a + b + c$

$= 21 + 15 + 24 = 60$ cm

Choose the correct answer

In the acute-angled triangle ABC , a = 8 cm. , b = 5 cm. , m (∠ C) = 60°
, then m (∠ A) ≃ ……………

(a) 83° 42̀ 12̏ (b) 81° 47̀ 12̏ (c) 38° 11̀ (d) 60° 23̀ 10̏

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$C = \sqrt{(8)^2 + (5)^2 - 2(8)(5)(\cos 60)} = 7$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{(5)^2 + (7)^2 - (8)^2}{2(5)(7)} = \frac{1}{7}$$

∴ m (∠A) = 81° 47̀ 12̏

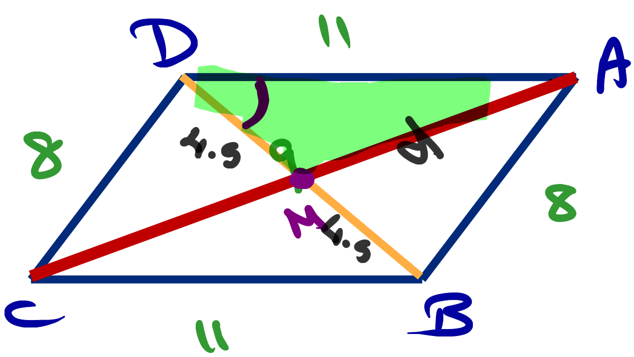
Choose the correct answer

ABCD is a parallelogram in which AB = 8 cm. , BC = 11 cm. , BD = 9 cm. , then the length of $\overline{AC}$ = ... cm.

(a) 9 (b) 10 (c) 11 (d) 17

In $\triangle ABD$

$$\cos D = \frac{a^2+b^2-d^2}{2ab}$$



$$\cos D = \frac{(9)^2+(11)^2-(8)^2}{2(9)(11)}$$

$$= \frac{23}{33}$$

In $\triangle ADM$

$$d^2 = m^2 + a^2 - 2ma\cos(\angle ADM)$$

$$AM = d = \sqrt{(11)^2+(4.5)^2-2(11)(4.5)\left(\frac{23}{33}\right)} = 8.5$$

$$AC = 2AM = 2 \times 8.5$$

$$= 17$$

Choose the correct answer

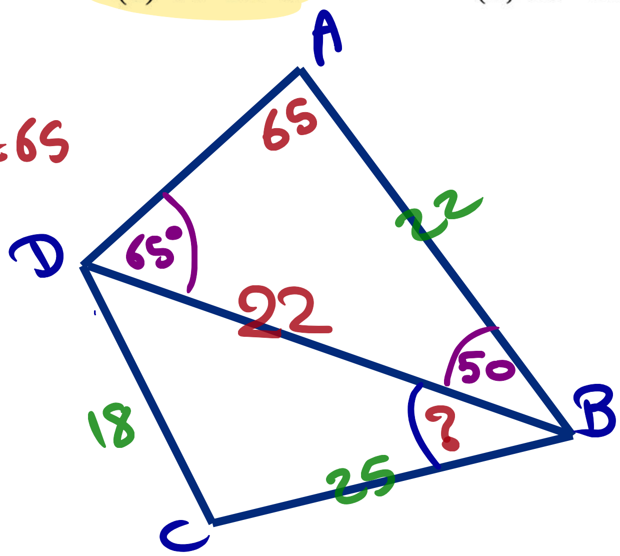
ABCD is quadrilateral in which AB = 22 cm. , BC = 25 cm. , DC = 18 cm. , m (∠ ADB) = 65° , m (∠ DBA) = 50° , then m (∠ CBD) ≃ ……………

(a) $80^\circ\ 75'$ (b) $42^\circ\ 49'\ 19''$ (c) $44^\circ\ 28'\ 6''$ (d) $85^\circ\ 30'$

In △ABD

$\because m(\angle A) = m(\angle BDA) = 65$

$\therefore AB = BD = 22 \text{ cm}$



In △BCD

$$\cos B = \frac{c^2 + d^2 - b^2}{2cd}$$

$$\cos B = \frac{(22)^2 + (25)^2 - (18)^2}{2(22)(25)} = \frac{157}{220}$$

$m(\angle B) = 44^\circ\ 28'\ 6''$

Choose the correct answer

ABC is a triangle in which $a = \sqrt{2}$ cm. , $b = \sqrt{3}$ cm. , $c = 2$ cm.

, then $\frac{\cos A \cos B}{\cos (A+B)} = \frac{\cos A \cos B}{-\cos C} = \frac{\frac{5}{4\sqrt{3}} \times \frac{3}{4\sqrt{2}}}{-\frac{1}{2\sqrt{6}}} = -\frac{15}{8}$

(a) $\frac{8}{15}$ (b) $\frac{-15}{8}$ (c) $\frac{-17}{15}$ (d) $\frac{8}{17}$

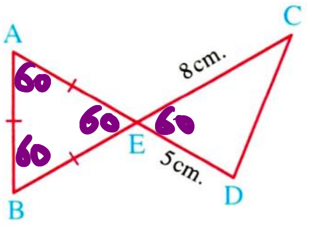
$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{3+4-2}{2(\sqrt{3})(2)} = \frac{5}{4\sqrt{3}}$

$\cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{4+2-3}{2(2)(\sqrt{2})} = \frac{3}{4\sqrt{2}}$

$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{2+3-4}{2(\sqrt{2})(\sqrt{3})} = \frac{1}{2\sqrt{6}}$

Choose the correct answer

In the opposite figure :



e

CD = ... cm.

(a) 6 (b) 7

(c) 8 (d) 9

$$e^2 = c^2 + d^2 - 2cd \cos E$$

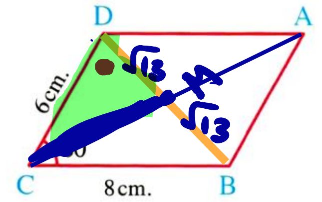
$$e = \sqrt{(5)^2 + (8)^2 - 2(5)(8)\cos 60} \simeq 7$$

Choose the correct answer

In the opposite figure :

ABCD is a parallelogram

, then AC = ... cm.



(a) $2\sqrt{13}$ (b) $2\sqrt{37}$

(c) $2\sqrt{17}$ (d) 148

In $\triangle BCD$

$BD = c = \sqrt{b^2 + d^2 - 2bd\cos C}$

$= \sqrt{(6)^2 + (8)^2 - 2(6)(8)\cos 60}$

$= 2\sqrt{13}$

$\cos D = \dfrac{b^2 + c^2 - d^2}{2bc}$

$= \dfrac{36 + 4 \times 13 - 64}{2(6)(2\sqrt{13})}$

$= \dfrac{\sqrt{13}}{13}$

In $\triangle DCM$

$d = \sqrt{c^2 + m^2 - 2cm\cos D}$

$= \sqrt{13 + 36 - 2(\sqrt{13})(6)\left(\frac{\sqrt{13}}{13}\right)}$

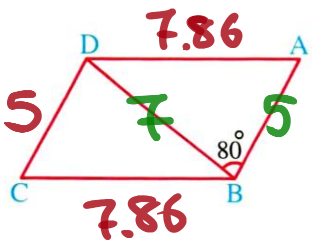
$= \sqrt{37}$

$\Rightarrow AC = 2d$

$= 2\sqrt{37}$

Choose the correct answer

In the opposite figure :



ABCD is a parallelogram

m (∠ ABD) = 80° , BD = 7 cm.

AB = 5 cm. , then the perimeter

of parallelogram = ·············· to the nearest cm.

(a) 25 (b) 26 (c) 29 (d) 30

In $\triangle ABD$

$$b^2 = d^2 + a^2 - 2da\cos B$$

$$b = \sqrt{(5)^2 + (7)^2 - 2(5)(7)\cos 80} \simeq 7.86$$

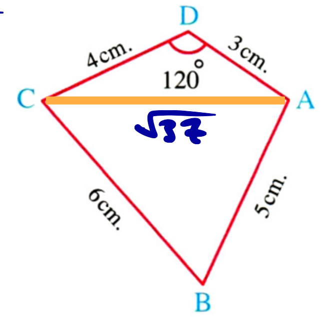
P. of ▱ABCD = $2(5 + 7.86) \simeq 26$

Choose the correct answer

In the opposite figure : In $\triangle ACD$

cos B = ...

$AC = d = \sqrt{a^2+c^2-2ac\cos D}$



(a) $\frac{1}{5}$

(b) $\frac{2}{5}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

$AC = \sqrt{16+9-2(4)(3)\cos 120}$

$= \sqrt{37}$

In $\triangle ABC$

$$\cos B = \frac{c^2+a^2-b^2}{2ca}$$

$$= \frac{25+36-37}{2(5)(6)} = \frac{2}{5}$$

Choose the correct answer

In the opposite figure :

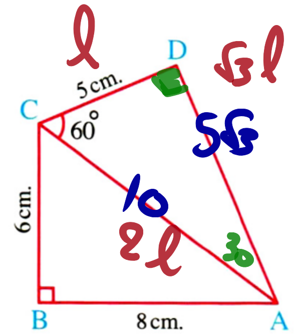
ABCD is a quadrilateral in which AB = 8 cm.

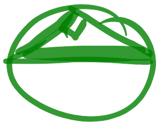
, BC = 6 cm. , m (∠ B) = 90°

, DC = 5 cm. and m (∠ ACD) = 60°

, then the area of the circumcircle of

the triangle ADC = ... cm².





(a) 9 π (b) 16 π (c) 25 π (d) 49 π

$$C = \sqrt{d^2 + a^2 - 2da\cos C}$$

$$AD = \sqrt{(10)^2 + (5)^2 - 2(10)(5)\cos 60}$$

$$= 5\sqrt{3}$$

Diameter = 10 cm

$r = 5$ cm

$A = \pi r^2$

$= 25\pi$ cm²

Choose the correct answer

In the opposite figure :

ABCD is a rectangle in which

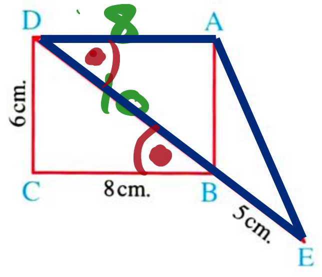
DC = 6 cm. , BC = 8 cm.

and E $\in \overrightarrow{DB}$ where BE = 5 cm.

, then AE = ………… cm.

$\cos(\angle ADB) = \frac{8}{10}$

$= \frac{4}{5}$



(a) $\sqrt{93}$ (b) $\sqrt{97}$ (c) 10 (d) $\sqrt{103}$

In $\triangle$ ADE

$AE = d = \sqrt{e^2 + a^2 - 2ea\cos(\angle ADB)}$

$= \sqrt{(8)^2 + (15)^2 - 2(8)(15)\left(\frac{4}{5}\right)} = \sqrt{97}$

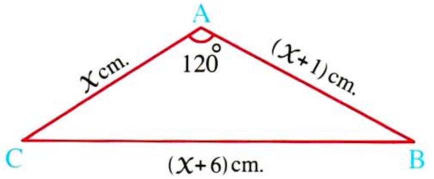
Choose the correct answer

In the opposite figure :

The value of X = ... cm.

(a) 7 (b) 8

(c) 9 (d) 10



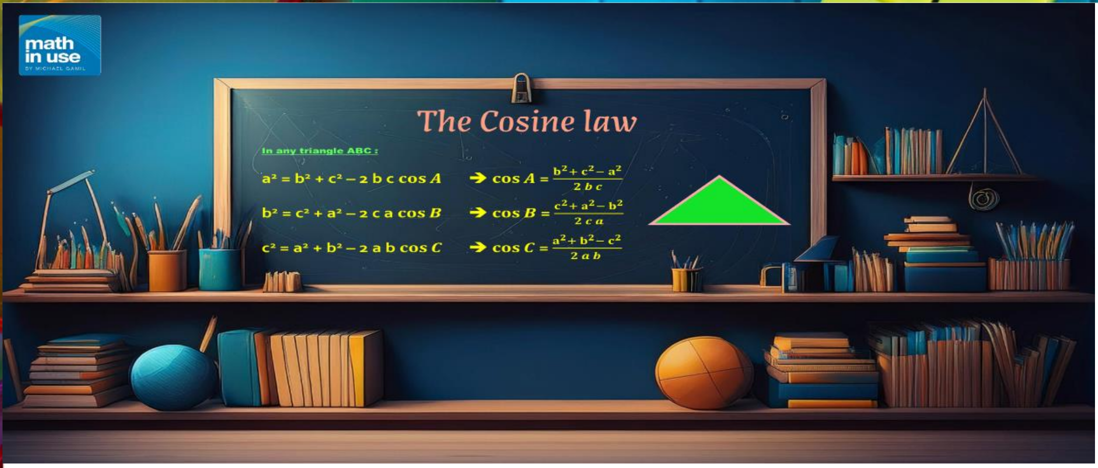
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$(x+6)^2 = x^2 + (x+1)^2 \cancel{-} + \cancel{2}x(x+1)\cos \cancel{120}$$

$$\cancel{x^2} + 12x + 36 = \cancel{x^2} + x^2 + 2x + 1 + x^2 + x$$

$$2x^2 - 9x - 35 = 0$$

$$x = 7 \qquad x = -\frac{7}{2} \text{ ref.}$$



Exercise 2

The Cosine rule

Answer each of the following questions

① XYZ is a triangle in which : m (∠ Z) = 95° , x = 13 cm. , y = 16 cm. Find z « 21.5 cm. »

$$Z^2 = x^2 + y^2 - 2xy\cos Z$$

$$Z = \sqrt{(13)^2 + (16)^2 - 2(13)(16)\cos 95} \simeq 21.5$$

Answer each of the following questions

(2) Find the measure of the smallest angle in ΔXYZ , where $x = 18$ cm. , y = 27 cm. and z = 24 cm. Find also the area of the circumcircle of ΔXYZ « 40° 48' , 596 cm^2 »

$\angle X$ is the Smallest angle

$$\cos X = \frac{y^2 + z^2 - x^2}{2yz}$$

$$= \frac{(27)^2 + (24)^2 - (18)^2}{2(27)(24)} = \frac{109}{144}$$

$$m(\angle X) = \cos^{-1}\left(\frac{109}{144}\right) = 40° 48'$$

$$\frac{x}{2 \sin x} = r \Rightarrow \frac{18}{2 \sin(40° 48')} = r$$

$$r \cong 13.77 \text{ cm}$$

$$A = \pi r^2 = \pi (13.77)$$

$$\simeq 596 \text{ cm}^2$$

Answer each of the following questions

3 The perimeter of the triangle ABC is 52 cm. , a = 13 cm. and b = 17 cm.
Find the measure of the greatest angle in the triangle , then find the area of the triangle to the nearest centimetre square. « 93° 22̀ , 110 cm.^{2} »

$$P = a + b + c = 52$$

$$13 + 17 + c = 52$$

$$\therefore c = 52 - 30 = 22 \text{ cm}$$

$\angle C$ is the greatest angle

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{(13)^2 + (17)^2 - (22)^2}{2(13)(17)} = -\frac{1}{17}$$

$$m(\angle C) = \cos^{-1}\left(-\frac{1}{17}\right) = 93^\circ \, 22' \, 20''$$

$$\text{A. of } \triangle ABC = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(13)(17) \sin 93^\circ \, 22' \, 20''$$

$$\simeq 110 \text{ cm}^2$$

Answer each of the following questions

(4) XYZ is a triangle in which sin X : sin Y : sin Z = 7 : 8 : 12

Find the measure of its greatest angle. « 106° 4̀ »

$\angle Z$ is the greatest angle

$\because \sin X : \sin Y : \sin Z$

$= x : y : z = 7 : 8 : 12$

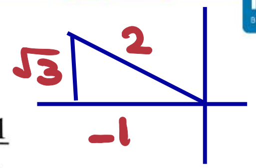
$\therefore x = 7m \quad y = 8m \quad z = 12m$

$$\cos Z = \frac{x^2 + y^2 - z^2}{2xy}$$

$$= \frac{(7m)^2 + (8m)^2 - (12m)^2}{2(7m)(8m)} = \frac{-31}{112}$$

$$m(\angle Z) = \cos^{-1}\left(\frac{-31}{112}\right) = 106^\circ\ 4'$$

Answer each of the following questions



(5) ABC is a triangle in which : a = 4 cm. , b = 5 cm. and $\cos C = \frac{-1}{2}$

Find c and the area of Δ ABC

$\sin C = \frac{\sqrt{3}}{2}$ « 7.8 cm. , $5\sqrt{3}$ cm.^{2} »

$m(\angle C) = 120$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c = \sqrt{16 + 25 - \cancel{2}(4)(5)(-\frac{1}{\cancel{2}})} = \sqrt{61} \simeq 7.8 \text{ cm}$$

$$\text{A. of } \Delta ABC = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(4)(5)\sin 120$$

$$= 5\sqrt{3} \text{ cm}^2$$

$$\simeq 8.66 \text{ cm}^2$$

Answer each of the following questions

6 ABC is a triangle in which $\frac{1}{3}$ sin A = $\frac{1}{4}$ sin B = $\frac{1}{5}$ sin C Find m (∠ C) and if the perimeter of the triangle = 24 cm. find its area. « 90° , 24 cm^2 »

$$\because \frac{\sin A}{3} = \frac{\sin B}{4} = \frac{\sin C}{5}$$

$$\therefore a : b : c = 3 : 4 : 5$$

$$\therefore a = 3m \quad b = 4m \quad c = 5m$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{(3m)^2 + (4m)^2 - (5m)^2}{2(3m)(4m)} = 0$$

$$\therefore m(\angle C) = 90^\circ$$

$$a + b + c = 3m + 4m + 5m = 24$$

$$12m = 24 \Rightarrow \boxed{m = 2}$$

$$a = 6\text{ cm} \quad b = 8\text{ cm} \quad c = 10\text{ cm}$$

$$A. \text{ of } \triangle ABC = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(6)(8) \sin 90$$

$$= 24 \text{ cm}^2$$

Answer each of the following questions

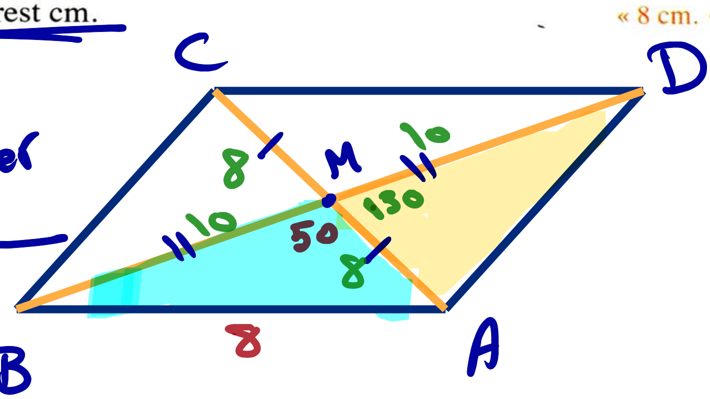
(7) ABCD is a parallelogram in which : AC = 16 cm. , DB = 20 cm. and m (∠ AMB) = 50° , where M is the point of intersection of its diagonals.

Find AB and AD to the nearest cm. « 8 cm. , 16 cm. »

Two diagonals bisect each other

$\therefore AM = CM = \frac{16}{2} = 8$ cm

$\therefore BM = DM = \frac{20}{2} = 10$ cm



In $\triangle$ AMB

$$(AB)^2 = m^2 = a^2 + b^2 - 2ab\cos M$$

$$m = \sqrt{(10)^2 + (8)^2 - 2(10)(8)\cos 50} \simeq 8 \text{ cm}$$

In $\triangle$ AMD

$$(AD)^2 = m^2 = a^2 + d^2 - 2ad\cos\angle AMD$$

$$\therefore AD = \sqrt{(10)^2 + (8)^2 - 2(10)(8)\cos(130)}$$

$$\simeq 16 \text{ cm}$$

Answer each of the following questions

(8) If the perimeter of the parallelogram ABCD is 20 cm. , the ratio between the two adjacent side lengths is 2 : 3 and BD = 8 cm. , then find the length of $\overline{AC}$ « 6.3 cm. »

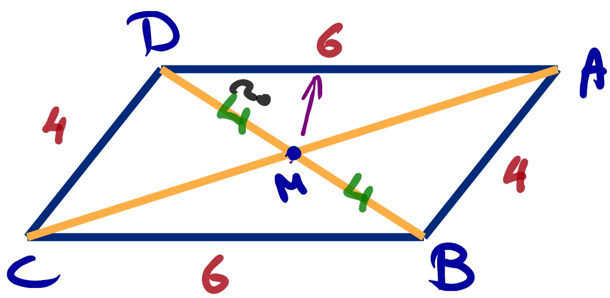
let AB = 2m , BC = 3m

AB + BC = 10

2m + 3m = 10

5m = 10 $\boxed{\therefore m = 2}$

AB = 4 cm BC = 6 cm



In △ ADB

$$\cos(\angle ADB) = \frac{b^2 + a^2 - d^2}{2ba}$$

$$\cos D = \frac{(6)^2 + (8)^2 - (4)^2}{2(6)(8)} = \frac{7}{8}$$

$$m(\angle ADB) = \cos^{-1}\left(\frac{7}{8}\right) = 28^\circ\, 57'$$

In △ ADM

$$(AM)^2 = d^2 = a^2 + m^2 - 2am\cos D$$

$$\therefore AM = \sqrt{16 + 36 - 2(4)(6)\cos D}$$

$$= \sqrt{10} \simeq 3.16 \text{ cm}$$

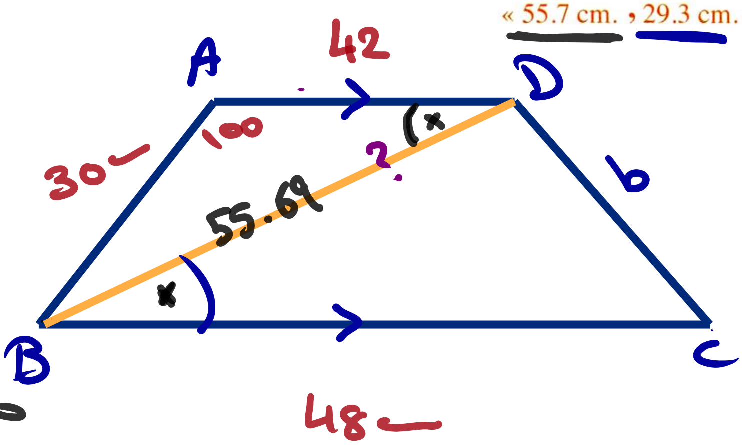
$$\therefore AC = 2AM = 2\sqrt{10}$$

$$\simeq 6.3 \text{ cm}$$

Answer each of the following questions

9. ABCD is a trapezium in which : $\overline{AD} \parallel \overline{BC}$, AD = 42 cm. , AB = 30 cm. , BC = 48 cm. and m (∠ A) = 100° **Find the length of each of :** $\overline{BD}$, $\overline{CD}$

« 55.7 cm. , 29.3 cm. »



In $\triangle ABD$

$(BD)^2 = a^2$

$= b^2 + d^2 - 2bd\cos A$

$BD = \sqrt{(42)^2 + (30)^2 - 2(42)(30)\cos 100}$

$\simeq 55.69$

$\cos D = \dfrac{a^2 + b^2 - d^2}{2ab}$

$= \dfrac{(55.69)^2 + (42)^2 - (30)^2}{2(55.69)(42)}$

$= 0.8477$

$\cos(ADB) = m(\angle DBC)$

$= 0.8477$

In $\triangle BCD$

$(CD)^2 = b^2$

$= c^2 + d^2 - 2cd\cos B$

$= (55.69)^2 + (48)^2 - 2(55.69)(48)\cos B$

$= 873.5$

$CD = \sqrt{873.5} \simeq 29.55$

Answer each of the following questions

(10) ABCD is a cyclic quadrilateral in which AB = AD = 9 cm. , BC = 5 cm. , CD = 8 cm.

Find : AC « 11 cm. »

Answer each of the following questions

(11) ABC is a triangle in which : $a = 5$ cm. , $m(\angle B) = 120°$ and its area is $10\sqrt{3}$ cm².

Find each of c and b and also $m(\angle A)$ « 8 cm. , 11.36 cm. , 22° 24̀ » math in use BY MICHAEL GAMIL

Answer each of the following questions

(12) If $\sin A = \frac{2}{3} \sin B = \frac{1}{2} \sin C$, $c - a = 4$ cm.

, **find each of :** b and m (∠ A) « 6 cm. , 28° 57̀ »

Answer each of the following questions

(13) ABC is a triangle whose perimeter is 34 cm. , $a = 12$ cm. and $b - c = 6$ cm.

Find the measure of its smallest angle , then calculate its area. « $34^\circ\, 46'\, 19''$, 47.9 cm^2 »

Answer each of the following questions

(14) ABC is a triangle in which $(a + b + c)(a + b - c) = k\,a\,b$

prove that : $k \in \,]0, 4[$, then **find :** $m(\angle C)$ when $k = 1$ « 120° »

Answer each of the following questions

15 ABC is a triangle in which : $b^2 = (c - a)^2 + c\,a$ **Find :** m (∠ B) « 60° »

Answer each of the following questions

(16) In Δ ABC : $\cos B = \frac{c}{2a}$ **, prove that :** Δ ABC is an isosceles triangle.

Answer each of the following questions

(17) In the parallelogram ABCD **, prove that :** $(AC)^2 + (BD)^2 = 2\,(AB)^2 + 2\,(BC)^2$