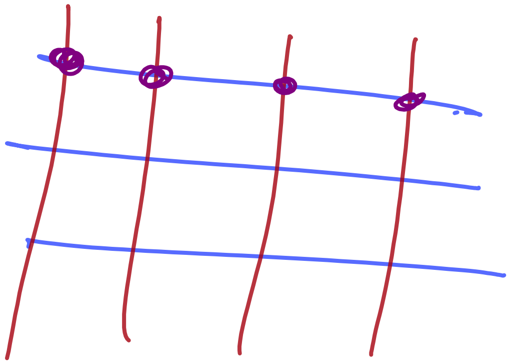
Final Revision Algebra

Choose the correct answer

If the matrix A of order 3×4 , then the row contains .... elements.

(a) 3 (b) 4 (c) 7 (d) 12



If A is a matrix of order 3×1 , B is a matrix of order 1×3 , then : AB is a matrix of order ....

(a) 3×1 (b) 1×1 (c) 3×3 (d) 1×3

$A_{3\times1} \times B_{1\times3} = C_{3\times3}$

If $A = \begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$, $mA = \begin{pmatrix} 0 & 3c \\ 2b & 24 \end{pmatrix}$, then $c + b - m =$ $-4 - 9 - (-6) = -7$

(a) -19 (b) -7 (c) 7 (d) 16

$$m\begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} 0 & 3c \\ 2b & 24 \end{pmatrix}$$

$-4m = 24 \Rightarrow m = -6$

$2m = 3c \Rightarrow 3c = -12$ $\therefore c = -4$

$3m = 2b \Rightarrow 2b = -18$ $b = -9$

If $x\begin{pmatrix}1\\3\end{pmatrix}+y\begin{pmatrix}2\\0\end{pmatrix}=\begin{pmatrix}5\\3\end{pmatrix}$, then $x+y=$ .... $1+2=3$

(a) 2 (b) 3 (c) 4 (d) 6

$$\begin{pmatrix}x\\3x\end{pmatrix}+\begin{pmatrix}2y\\0\end{pmatrix}=\begin{pmatrix}5\\3\end{pmatrix}$$

$x+2y=5$

$1+2y=5$

$2y=4$

$y=2$

$3x=3$

$x=1$

If $A = \begin{pmatrix} 3 & -2 & 7 \\ 5 & -4 & 2 \end{pmatrix}$, and $B = A^t$, then $a_{13} + b_{31} = 7 + 7 = 14$

(a) 4 (b) 9 (c) 14 (d) 10

$B = A^t = \begin{pmatrix} 3 & 5 \\ -2 & -4 \\ 7 & 2 \end{pmatrix}$

If X is a matrix , then $(X^t)^t + X =$ ……… $X + X = 2X$

(a) X (b) 2 X (c) $\square$ (d) zero

$(X^t)^t - X =$

$X - X = \square$

Note that

If the matrix A of order 2×2 where $A = (a_{ij})$, where $a_{ij} = 2i + j$, then A = ...

(a) $\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & 5 \\ 6 & 4 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 6 \\ 5 & 4 \end{pmatrix}$

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$a_{ij} = 2i + j$ V. imp

$a_{11} = 2(1) + 1 = 3$

$a_{12} = 2(1) + 2 = 4$

$a_{21} = 2(2) + 1 = 5$

$a_{22} = 2(2) + 2 = 6$

$A = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$

If $A = \begin{pmatrix} x & y \\ z & \ell \end{pmatrix}$ is skew symmetric matrix , then $\frac{y}{z} + x - \ell =$ ....

(a) 1 (b) 0 (c) – 1 (d) – 2

$x = \ell = 0$

$y = -z$

$\frac{-z}{z} + 0 - 0$

$= -1$

If $A = \begin{pmatrix} 1 & 1 & x-1 \\ 1 & 3 & 5 \\ -1 & 5 & 6 \end{pmatrix}$ is a symmetric matrix , then $x =$ ……………

(a) -1 (b) zero (c) 4 (d) 6

$x - 1 = -1$

$x = \text{zero}$

If $A + A^t = O$, then A is a .... matrix.

$A = A^t$ (Sym)
$A = -A^t$ (Skew)

(a) row (b) column (c) symmetric (d) skew

$A + A^t = O$

$A = -A^t$

$$\boxed{\begin{gathered} A - A^t = O \\ A = A^t \\ \text{sym.} \end{gathered}}$$

If $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 3\,I$ where I is the unit matrix , then a + b + c + d = ...6......

$3+0+0+3$

(a) 5 (b) 6 (c) 7 (d) 8

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$a=3$ $b=0$ $c=0$ $d=3$

If $\begin{pmatrix} 3^x & 2x+y \\ x+y & 3^y \end{pmatrix} = \begin{pmatrix} 25 & b \\ a & 5 \end{pmatrix}$, then $\frac{b}{a} = $ ....

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) 15 (d) 5

$3^x = 25$

$3^x = 5^2 \rightarrow 3^y = 5$

$3^x = (3^y)^2$

$3^x = 3^{2y} \Rightarrow x = 2y$

$b = 2x + y$

$b = 2(2y) + y$

$\boxed{b = 5y}$

$a = x + y$

$a = 2y + y$

$\boxed{a = 3y}$

$\therefore \frac{b}{a} = \frac{5y}{3y} = \frac{5}{3}$

If A , B are two matrices where AB = $\begin{pmatrix} 3 & -1 \\ 4 & 5 \end{pmatrix}$, then $B^t A^t$ = ...

(a) $\begin{pmatrix} 3 & -1 \\ 4 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 4 \\ -1 & 5 \end{pmatrix}$ (c) $\begin{pmatrix} 5 & -1 \\ 4 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 5 & 4 \\ -1 & 3 \end{pmatrix}$

$B^t A^t = (AB)^t$

$B^{-1} A^{-1} = (AB)^{-1}$

$B^t A^t = \begin{pmatrix} 3 & -1 \\ 4 & 5 \end{pmatrix}^t = \begin{pmatrix} 3 & 4 \\ -1 & 5 \end{pmatrix}$

If $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$, then $A^2 = \ldots\ldots\ldots\ldots\ldots$

(a) $\begin{pmatrix} 4 & 1 \\ 9 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 4 & 9 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & -3 \\ 9 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -3 \\ 9 & -2 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -3 \\ 9 & -2 \end{pmatrix}$$

If $\begin{pmatrix} 3 & 2 & x \\ 4 & 5 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ y & 2 \end{pmatrix} = \begin{pmatrix} 14 & -1 \\ 19 & -2 \end{pmatrix}$, then $x - y =$ ....

(a) 3 (b) – 3 (c) 5 (d) – 5

$-3 + 0 + 2x = -1$

$2x = -1 + 3$

$2x = 2$

$x = 1$

$8 + 5 + y = 19$

$y = 19 - 13$

$y = 6$

$x - y = 1 - 6 = -5$

If $A = \begin{pmatrix} 2 & x \\ -x & 2 \end{pmatrix}$, $A^2 = \begin{pmatrix} -5 & 12 \\ -12 & -5 \end{pmatrix}$, then $x =$ ....

(a) 3 (b) – 3 (c) 4 (d) zero.

$$A^2 = \begin{pmatrix} 2 & x \\ -x & 2 \end{pmatrix}\begin{pmatrix} 2 & x \\ -x & 2 \end{pmatrix} = \begin{pmatrix} -5 & 12 \\ -12 & -5 \end{pmatrix}$$

$$4 - x^2 = -5$$

$$-x^2 = -9$$

$$x^2 = 9$$

$$x = \pm 3$$

$$2x + 2x = 12$$

$$4x = 12$$

$$\boxed{x = 3}$$

$\begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}\begin{pmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{pmatrix} = \dots\dots\dots\dots$

(a) I (b) – I (c) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} \sin^2\theta + \cos^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & -\cos^2\theta - \sin^2\theta \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

If $\begin{vmatrix} 3+\sin\theta & 1-\cos\theta \\ 4 & \sin\theta \end{vmatrix}$ = zero , then the value of θ = ...

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

Try & error

$3\sin\theta + \sin^2\theta - 4 + 4\cos\theta = 0$

$3\sin\frac{\pi}{2} + \sin^2\frac{\pi}{2} - 4 + 4\cos\frac{\pi}{2}$

$3 + 1 - 4 + 0$ = zero Satisfy

If $\begin{vmatrix} x & -1 \\ 2 & x \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 2 & x \end{vmatrix} = 2$, then $x =$ ....

(a) 3 or – 2

(b) – 3 or 2

(c) 3 or 2

(d) – 3 or – 2

$x^2 + 2 + x - 6 = 2$

$x^2 + x - 6 = 0$

$x = 2$ $\quad$ $x = -3$

If $\dfrac{x}{\begin{vmatrix} 3 & 7 \\ 5 & 6 \end{vmatrix}} = \dfrac{y}{\begin{vmatrix} 7 & 4 \\ 6 & 1 \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} 3 & 4 \\ 5 & 1 \end{vmatrix}}$, then $x + y =$ ......$1+1$...... $= 2$

(a) -2 (b) -1 (c) 1 (d) 2

$$\frac{x}{-17} = \frac{y}{-17} = \frac{1}{-17}$$

$$x = y = 1$$

If $\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} \times \begin{pmatrix} x & 2y \\ 0 & z \end{pmatrix} + I = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}^t$, then $x \times y \times z =$ ....

(a) 1 (b) – 1 (c) 2 (d) 4

$$\begin{pmatrix} -x & -2y \\ 0 & -z \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} -x+1 & -2y \\ 0 & -z+1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$$

$-x+1=3$ $-2y=1$ $-z+1=2$

$-x=2$ $\boxed{y=\frac{-1}{2}}$ $-z=1$

$\boxed{x=-2}$ $\boxed{z=-1}$

$$xyz = -2 \times -\frac{1}{2} \times -1 = -1$$

math in use BY MICHAEL GAMIL

If $\begin{vmatrix} 2x & 0 & 0 \\ 1 & 3x & 0 \\ 2 & 4 & -x \end{vmatrix} = 48$, then the value of $x =$ ....

(a) 2 (b) – 2 (c) 3 (d) – 3

$-6x^3 = 48$

$x^3 = \frac{48}{-6} \Rightarrow x^3 = -8$

$x = \sqrt[3]{-8} = -2$

If $\begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 12$, then $\begin{vmatrix} a & d & x \\ b & e & y \\ c & f & z \end{vmatrix} =$ 12

(a) -12 (b) 12 (c) zero (d) 24

Rem. $|A^t| = |A|$

If $\begin{vmatrix} x & y & z \\ a & b & c \\ 2 & -1 & 8 \end{vmatrix} = 10$, then $\begin{vmatrix} 2a & 2b & 2c \\ x & y & z \\ 4 & -2 & 16 \end{vmatrix} = \cdots\cdots\cdots$

(a) – 40 (b) – 20 (c) – 10 (d) 0

$$2 \times 2 \begin{vmatrix} a & b & c \\ x & y & z \\ 2 & -1 & 8 \end{vmatrix}$$

exchange $R_1 \Leftrightarrow R_2$

$$-2 \times 2 \begin{vmatrix} x & y & z \\ a & b & c \\ 2 & -1 & 8 \end{vmatrix} = -2 \times 2 \times 10$$

$$= -40$$

If A (3 , 5) , B (2 , 0) and C (– 3 , 3) k

, then the area of Δ ABC equals .... square units.

(a) 28 (b) 14 (c) 7 (d) 2

$$A\text{ of } \triangle ABC = \frac{1}{2}\left|\begin{vmatrix} 3 & 5 & 1 \\ 2 & 0 & 1 \\ -3 & 3 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}(28) = 14 \text{ S.u}$$

if A , B & C are Collinear

$$\therefore \begin{vmatrix} & \end{vmatrix} = \text{zero}$$

If the area of the triangle whose vertices are (k , 0) , (4 , 0) , (0 , 2) equals 4 square units , then k = ...

(a) zero or 8 (b) – 4 or 4 (c) zero or – 8 (d) 8 or – 8

$$A = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 4$$

$$\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 8$$

$$k(-2) + 1(8) = 8$$

$$-2k + 8 = 8 \qquad\qquad -2k + 8 = -8$$

$$-2k = 0 \qquad\qquad -2k = -8 - 8$$

$$\boxed{k = 0} \qquad\qquad -2k = -16$$

$$\boxed{k = 8}$$

If L , M are the two roots of the equation : $x^2 - 7x + 2 = 0$, then the value of $\begin{vmatrix} L^2 M & -L^2 \\ M^2 & M \end{vmatrix} = \ldots\ldots\ldots\ldots\ldots$

(a) zero (b) 4 (c) 8 (d) 16

$L^2M^2 + L^2M^2$

$= 2L^2M^2$

$= 2(LM)^2$

$= 2(2)^2 = 2 \times 4 = 8$

$a = 1 \quad b = -7 \quad c = 2$

$\text{Sum} = L + M = \frac{-b}{a} = 7$

$\text{Product} = LM = \frac{c}{a} = 2$

If A is a square matrix of order 2×2 and $|2A| = 8$, then $|3A| = \ldots\ldots\ldots\ldots\ldots$

(a) 9 (b) 12 (c) 18 (d) 24

$|2A| = 2^2 |A| = 8$

$4|A| = 8 \Longrightarrow |A| = 2$

$|3A| = 3^2 |A| = 9|A| = 9 \times 2$

$= 18$

The system of equations $a_1 x + b_1 y = c_1$, $a_2 x + b_2 y = c_2$

If $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 5$, $\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = -10$, $\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = 15$

, then $(x , y) =$ ……………

(a) $(-2 , 3)$ (b) $(3 , -2)$ (c) $(-50 , 75)$ (d) $(75 , -50)$

$\Delta = 5$ $\Delta_x = -10$ $\Delta_y = 15$

$x = \frac{\Delta x}{\Delta} = \frac{-10}{5} = -2$, $y = \frac{\Delta y}{\Delta} = \frac{15}{5} = 3$

$(x, y) = (-2, 3)$

By solving the system of equations : $2x + 3y - z = 1$, $3x + 5y + 2z = 8$ and $x - 2y - 3z = -1$ the value of $\frac{\Delta_x}{\Delta} = \dots\dots\dots$

(a) -1 (b) 2 (c) -2 (d) 3

$$\Delta = \begin{array}{c} \begin{array}{ccc} x & & \end{array} \\ \begin{vmatrix} 2 & 3 & -1 \\ 3 & 5 & 2 \\ 1 & -2 & -3 \end{vmatrix} \end{array} = 22$$

$$\Delta_x = \begin{vmatrix} 1 & 3 & -1 \\ 8 & 5 & 2 \\ -1 & -2 & -3 \end{vmatrix} = 66$$

$$\frac{\Delta_x}{\Delta} = \frac{66}{22} = 3$$

If the matrix $\begin{pmatrix} x & 2 \\ 9 & x-3 \end{pmatrix}$ has no multiplicative inverse , then $x \in$ ……………

(a) $R - \{-3 , 6\}$ (b) $R - \{0 , 3\}$ (c) $\{0 , 3\}$ (d) $\{-3 , 6\}$

$\Delta = 0$

$\begin{vmatrix} x & 2 \\ 9 & x-3 \end{vmatrix} = 0$

$x^2 - 3x - 18 = 0$

$x = 6 \qquad x = -3$

It has a multi. inverse $\Delta \neq 0$

$x \in R - \{-3, 6\}$

If $X = \begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}$, then $X^{-1} = \ldots\ldots\ldots\ldots$

(a) $\begin{pmatrix} \frac{1}{2} & -2 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{2} & 2 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} \frac{1}{2} & -2 \\ 0 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

$$X^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -4 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -2 \\ 0 & 1 \end{pmatrix}$$

If $A = \begin{pmatrix} x & 2 \\ y & -2 \end{pmatrix}$, $A \times A^{-1} = A^2$, then $x \times y =$ ....

(a) -3 (b) -2 (c) 2 (d) 3

$A \times A^{-1} = I$

$A^2 = I$

$$\begin{pmatrix} x & 2 \\ y & -2 \end{pmatrix} \begin{pmatrix} x & 2 \\ y & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$2x - 4 = 0$

$2x = 4$

$\boxed{x = 2}$

$2y + 4 = 1$

$2y = -3$

$\boxed{y = -\frac{3}{2}}$

$x \times y = 2 \times \frac{-3}{2} = -3$

If $B = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}$ and AB = I , then A = ...

(a) $\frac{1}{10}\begin{pmatrix} -1 & 4 \\ 3 & -2 \end{pmatrix}$ (b) $\frac{-1}{10}\begin{pmatrix} -1 & 4 \\ 3 & -2 \end{pmatrix}$ (c) $\frac{-1}{10}\begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 7 & 4 \\ -2 & 6 \end{pmatrix}$

$AB = I$

$\therefore A = B^{-1}$

$A = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}^{-1}$

$A = \frac{1}{-10}\begin{pmatrix} 1 & -4 \\ -3 & 2 \end{pmatrix}$

$= \frac{1}{10}\begin{pmatrix} -1 & 4 \\ 3 & -2 \end{pmatrix}$

When solving the two equations : $a\,x + b\,y = 4$, $c\,x + d\,y = 2$ its found that the multiplicative inverse of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}$, then $2\,x + 3\,y =$ ……………

(a) 2 (b) 3 (c) 4 (d) – 2

V.imp

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}\begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \Longrightarrow \boxed{\begin{matrix} x = 2 \\ y = 0 \end{matrix}}$$

$$2x + 3y = 2(2) + 3(0) = 4$$

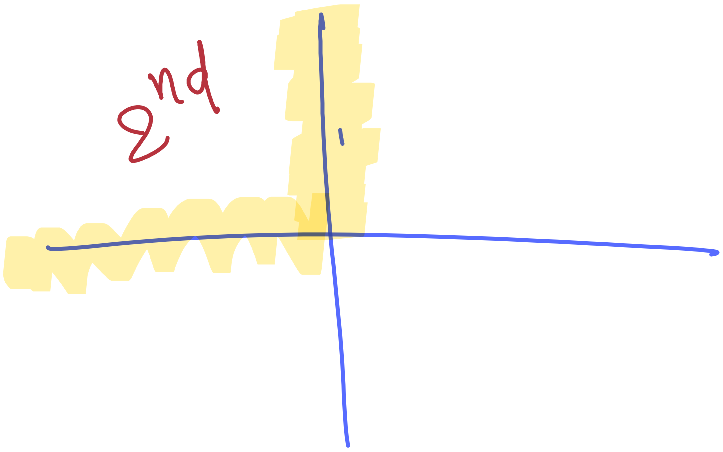
The point which belongs to the solution set of the inequality :

$x > 3$, $y < 1$, $x + y \leq 5$ is ……………

(a) (6 , – 2) ~~(b)~~ (1 , – 2) ~~(c)~~ (4 , 4) ~~(d)~~ (3 , – 2)

The region representing the solution set of the two inequalities : $y > 0$, $x < 0$ in $\mathbb{R} \times \mathbb{R}$ in the quadrant ....

(a) first. (b) second. (c) third. (d) fourth.



If the point $(-1 , 3)$ is belong to the solution set of the inequality $x + y \geq k$, then ....

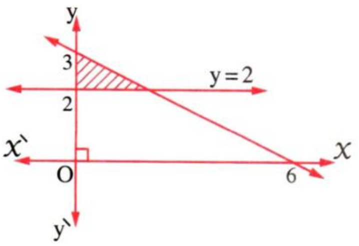
(a) $k \geq 2$ (b) $k \leq 2$ (c) $k > 2$ (d) $k < 2$

$-1+3 \geq k$

$2 \geq k$

$k \leq 2$

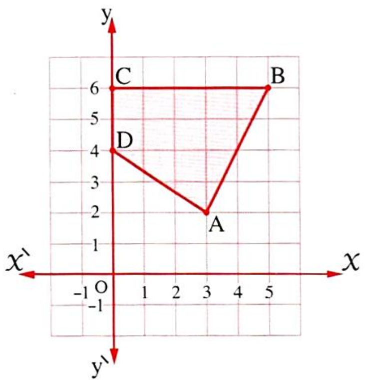
The shaded area in the opposite figure represents the solution set of : $x \geq 0$, $y \geq 2$ and ....



(a) $x + 2y - 6 \leq 0$ (b) $x + 2y + 6 \leq 0$

(c) $5x + 2y - 6 \geq 0$ (d) $x + 2y + 6 \geq 0$

The opposite figure represents the solution set of a system of inequalities , then the smallest value of the objective function $P = 3x + 2y$ is ……………



(a) 6
(b) 8
(c) 12
(d) 13

Best wishes
Mr. Michael Gamil