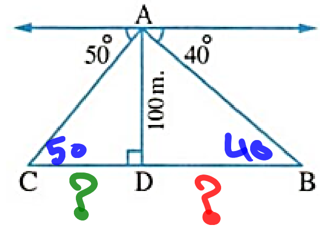
Trigonometry 2nd Sec.

Final revision

In the opposite figure , $\overline{AD}$ represents a light house of height 100 metre from the ground surface , a person from its top observed measure of depression angles of two boats «B , C» in the same hoirzontal plane passing through the light house base and in two opposite sides from it to be 40° and 50° repsectively , then distance between the two boats (BC) ≃ .... m.



(a) 314 (b) 152 (c) 203 (d) 232

In $\triangle ABD$

$\tan 40 = \frac{100}{BD}$

$BD = 119.18$

In $\triangle ACD$

$\tan 50 = \frac{100}{CD}$

$CD = 83.91$

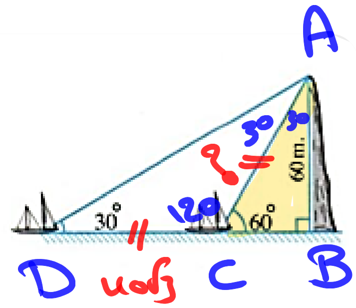
$\boxed{AC = CD}$

(6) In the opposite figure :

The distance between the two ships = ... m.

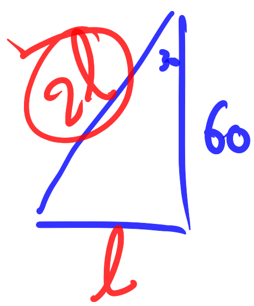
(a) 60 (b) $40\sqrt{3}$

(c) $\frac{60}{\sqrt{3}}$ (d) $60\sqrt{3}$



$$\frac{\sin 60}{1} = \frac{60}{AC} \Rightarrow AC = \frac{1 \times 60}{\sin 60}$$

$$= 40\sqrt{3}$$



$$(2l)^2 = l^2 + (60)^2$$

$$4l^2 - l^2 = 3600$$

$$3l^2 = 3600 \Rightarrow l^2 = 1200$$

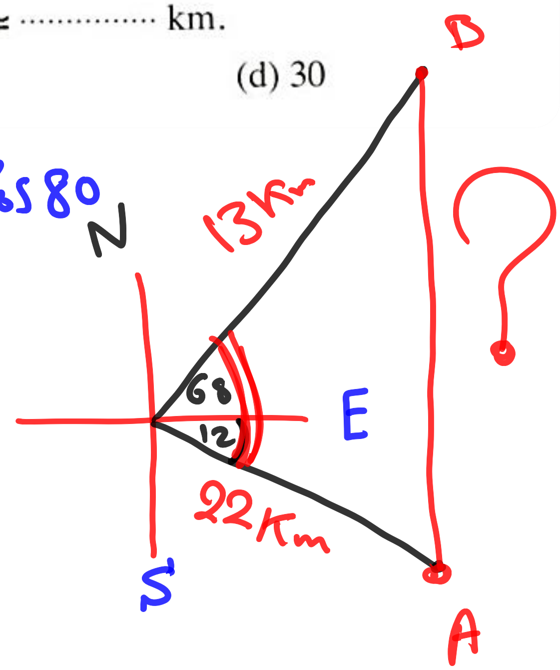
$$l = 20\sqrt{3}$$

$$2l = 2 \times 20\sqrt{3} = 40\sqrt{3}$$

A ship sailed from a certain point in the direction of 12° South of the East with velocity 11 km./h. At the same moment another ship sailed from the same point in the direction of 68° North of the East with velocity 6.5 km./h. , then the distance between the two ships after 2 hours from the time of their sailing together ≃ .... km.

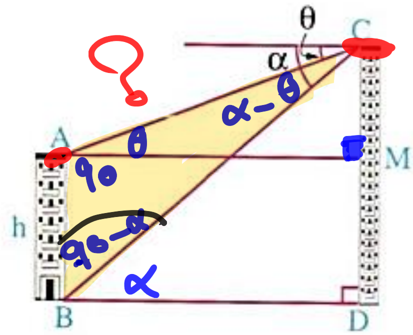
(a) 24 (b) 26 (c) 28 (d) 30

$$(AB)^2 = (22)^2 + (13)^2 - 2(22)(13)\cos 80$$



In the opposite figure :

$\overline{CD}$ represents a tower , $\overline{AB}$ represents a house of height h metre and θ , α are the measures of the depression angles of A and B from C respectively , then the distance between the top of the tower and the top of the house = ... m.



(a) $\dfrac{h \sin \alpha}{\sin \theta}$ (b) $h \sin \alpha \cos \theta$

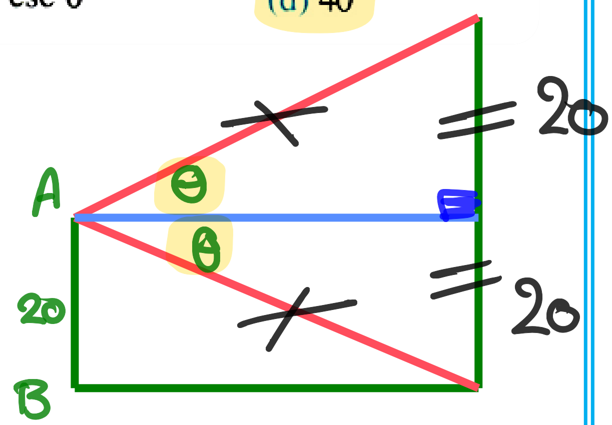
(c) $\dfrac{h \cos \theta}{\sin (\alpha - \theta)}$ (d) $\dfrac{h \cos \alpha}{\sin (\alpha - \theta)}$

$$\frac{h}{\sin(\alpha-\theta)} = \frac{AC}{\sin(90-\alpha)} \Rightarrow \frac{h}{\sin(\alpha-\theta)} = \frac{AC}{\cos\alpha}$$

$$AC = \frac{h \cos \alpha}{\sin(\alpha - \theta)}$$

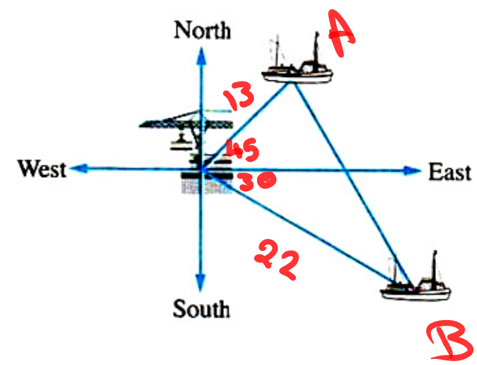
From a balcony of a house 20 m. high , a man observed that the top and the base of a tower in front of him have elevation angle and depression angle each of measure = θ , giving that the house and the tower are on the same horizontal plane , then the height of the tower = ... m.

(a) $20 \tan \theta$ (b) $\frac{20}{\tan \theta}$ (c) $40 \csc \theta$ (d) 40



In the opposite figure :

Two ships sailed from the same port and after 2 hours one of them became at a distance 13 km. in the direction north of the east of the port and the other became at a distance 22 km. in direction 30° south of the East of the port. , then at this instant the distance between the two ships ≈ .... km.



(a) 26 (b) 24 (c) 22 (d) 505

$$AB = \sqrt{(13)^2 + (22)^2 - 2(13)(22)\cos 75}$$

$$\simeq 22$$

If $\sin A = \frac{-3}{5}$, then $\cos 2A = \ldots\ldots\ldots\ldots\ldots$

(a) $\frac{16}{25}$ (b) $\frac{-16}{25}$ (c) $\frac{7}{25}$ (d) $\frac{-7}{25}$

$$\cos 2A = 1 - 2\sin^2 A$$

$$= 1 - 2\left(\frac{-3}{5}\right)^2 = \frac{7}{25}$$

If $\cos^4 x - \sin^4 x = \cos kx$, then k = ...

(a) 4 (b) 83 (c) 6 (d) 2

$$\cos^4 x - \sin^4 x = \underbrace{(\cos^2 x - \sin^2 x)}_{\cos 2x}\underbrace{(\cos^2 x + \sin^2 x)}_{1}$$

$$\cos 2x \times 1$$

$$= \cos 2x$$

$\cos 5x \cos 3x + \sin 5x \sin 3x =$ ....

(a) $\cos 2x$ (b) $\cos 8x$ (c) $\sin 8x$ (d) $\sin 2x$

$\cos(5x - 3x)$

$\cos 2x$

In Δ ABC : $\sin \frac{A+B}{2} \cos \frac{C}{2} + \cos \frac{A+B}{2} \sin \frac{C}{2} =$ ....

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) zero

$$\sin \left[\frac{A+B}{2} + \frac{C}{2} \right]$$

$$\sin \left[\frac{A+B+C}{2} \right] = \sin 90$$

$$= 1$$

If $\sin A + \cos A = \frac{7}{5}$ where A acute angle , then $\cos 2A =$ ……………

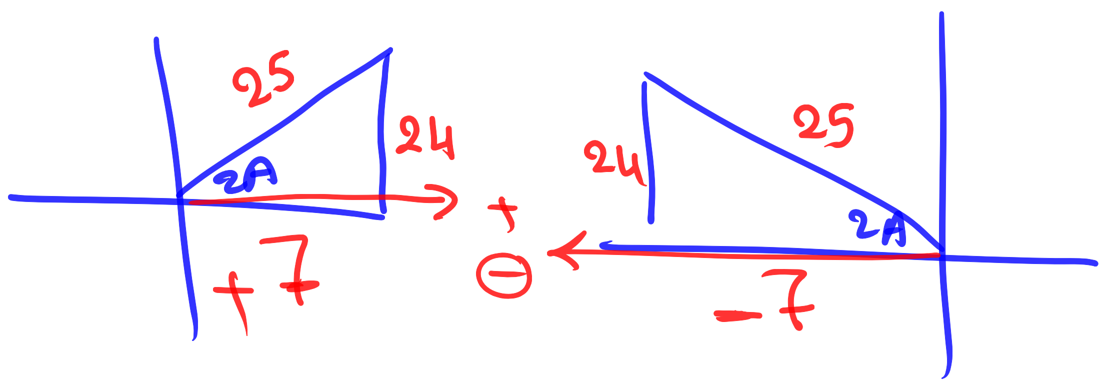
(a) $\frac{24}{25}$ (b) $\pm\frac{24}{25}$ (c) $\frac{7}{25}$ (d) $\pm\frac{7}{25}$

$$(\sin A + \cos A)^2 = \left(\frac{7}{5}\right)^2$$

$$\boxed{\sin^2 A + \cos^2 A} + \boxed{2 \sin A \cos A} = \frac{49}{25}$$

$$1 + \sin 2A = \frac{49}{25}$$

$$\sin 2A = \frac{24}{25}$$



$$\cos A = \pm \frac{7}{25}$$

$\frac{1+\cos 2x}{\sin 2x} = \frac{\cancel{1}+2\cos^2 x-\cancel{1}}{2\sin x\cos x} = \frac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\,\cancel{\cos x}}$

(a) $\tan x$ (b) $\cos x$ (c) $\sin x$ (d) $\cot x$

$$= \frac{\cos x}{\sin x} = \cot x$$

$$\frac{\tan 83^\circ - \tan 38^\circ}{1 + \tan 83^\circ \tan 38^\circ} = \dots\dots\dots\dots$$

(a) $\tan 45^\circ$ (b) $\tan 121^\circ$ (c) $\tan 22.5^\circ$ (d) $\tan 60.5^\circ$

$\tan(83 - 38)$

$\tan 45$

In ΔABC if $\tan A = 3$, $\tan B = 5$, then $\tan C =$ ....

(a) $\frac{-1}{7}$ (b) $\frac{1}{7}$ (c) $\frac{-4}{7}$ (d) $\frac{4}{7}$

$$A+B+C=180$$

$$A+B=180-C$$

$$\tan(A+B)=\tan(180-C)$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$$

$$\frac{3+5}{1-(3)(5)}=-\tan C$$

$$\cancel{-}\tan C=\cancel{-}\frac{4}{7}$$

$$\tan C=\frac{4}{7}$$

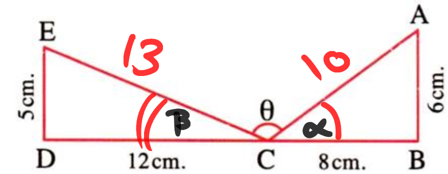
The area of the equilateral triangle whose side length equals x cm. equals .... cm^2.

(a) x^2 (b) $\frac{\sqrt{3}}{2}x^2$ (c) $\frac{\sqrt{3}}{4}x^2$ (d) $\frac{1}{2}x^2$

$$A = \frac{1}{2} x \cdot x \sin 60$$

$$= \frac{\sqrt{3}}{4} x^2$$

In the opposite figure :



$\sin\theta = \ldots\ldots\ldots\ldots\ldots$

(a) $\frac{56}{65}$ (b) $\frac{16}{65}$

(c) $\frac{64}{65}$ (d) $\frac{13}{12}$

$\alpha + \beta + \theta = 180$

$\alpha + \beta = 180 - \theta$

$\sin(\alpha + \beta) = \sin(180 - \theta)$

$\sin\alpha \cos\beta + \cos\alpha \sin\beta = \sin\theta$

$\left(\frac{6}{10}\right)\left(\frac{12}{13}\right) + \left(\frac{8}{10}\right)\left(\frac{5}{13}\right) = \sin\theta$

$\sin\theta = \frac{56}{65}$

$\cos^2 C - \cos 2C = \ldots\ldots\ldots\ldots\ldots$

(a) $\sin C$ (b) $\cos C$ (c) $\sin^2 C$ (d) $\tan C$

$\cos^2 C - [\cos^2 C - \sin^2 C]$

$\cancel{\cos^2 C} - \cancel{\cos^2 C} + \sin^2 C = \sin^2 C$

If $\cot \frac{A}{2} - \tan \frac{A}{2} = 4$, then $\tan A = \dfrac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}} = \dfrac{2(-2+\sqrt{5})}{1-(-2+\sqrt{5})^2} = \dfrac{1}{2}$

(a) $\frac{3}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$

$$\frac{1}{\tan\frac{A}{2}} - \tan\frac{A}{2} = 4 \qquad \times \tan\frac{A}{2}$$

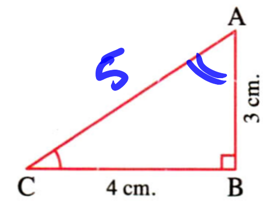
$$1 - \tan^2\frac{A}{2} = 4\tan\frac{A}{2}$$

$$\tan^2\frac{A}{2} + 4\tan\frac{A}{2} - 1 = 0$$

$$\tan\frac{A}{2} = -2+\sqrt{5} \quad \text{or} \quad -2-\sqrt{5}$$

In the opposite figure :

The sum of the lengths of the two radii of the inscribed circle and circumcircle of the Δ ABC equals .... cm.



(a) 4 (b) 3 (c) 5 (d) 3.5

$\frac{a}{\sin A} = 2r$

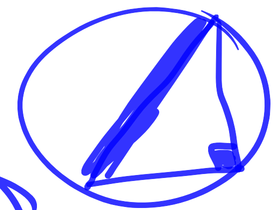
$\frac{\cancel{4}}{\frac{\cancel{4}}{5}} = 2r$

$5 = 2r$

$r_1 = 2.5$

$2r = 5$

$r_2 = 2.5$



$\sin x \cos y - \cos x \sin y$

If $\sin x - \cos y = \frac{1}{6}$, $\cos x + \sin y = \frac{5}{6}$, then $\sin(x - y) =$ ....

(a) $\frac{23}{36}$ (b) $\frac{19}{24}$ (c) $\frac{17}{23}$ (d) $\frac{25}{48}$

$$\boxed{\sin^2 x} + \boxed{\cos^2 y} - 2\sin x \cos y = \frac{1}{36}$$

$$\cos^2 x + \sin^2 y + 2\cos x \sin y = \frac{25}{36}$$

$$1 + 1 - 2[\sin x \cos y - \cos x \sin y] = \frac{13}{18}$$

$$2 - 2[\sin(x-y)] = \frac{13}{18}$$

$$2 - \frac{13}{18} = 2\sin(x-y)$$

$$\frac{23}{18} = 2\sin(x-y)$$

$$\sin(x-y) = \frac{23}{36}$$

If $\sin x + \cos x = \frac{\sqrt{2}}{3}$, then $\sin 2x =$ ....

(a) $\frac{-1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{-7}{9}$ (d) $\frac{\sqrt{3}}{2}$

$$(\sin x + \cos x)^2 = \left(\frac{\sqrt{2}}{3}\right)^2$$

$$\boxed{\sin^2 x + \cos^2 x} + \boxed{2\sin x \cos x} = \frac{2}{9}$$

$$1 + \sin 2x = \frac{2}{9}$$

$$\sin 2x = \frac{2}{9} - 1 = -\frac{7}{9}$$

If $\tan(\theta + 45°) = \frac{3}{2}$, then $\tan\theta = \ldots\ldots\ldots\ldots$

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{9}{4}$ (d) $\frac{2}{3}$

$$\frac{\tan\theta + \tan 45}{1 - \tan\theta \tan 45} = \frac{3}{2}$$

$$\frac{\tan\theta + 1}{1 - \tan\theta} = \frac{3}{2}$$

$$2\tan\theta + 2 = 3 - 3\tan\theta$$

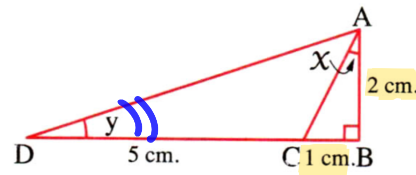
$$2\tan\theta + 3\tan\theta = 3 - 2$$

$$5\tan\theta = 1$$

$$\tan\theta = \frac{1}{5}$$

In the opposite figure :

ABD is right angled triangle at B , then $x + y = \ldots\ldots\ldots\ldots°$



(a) 30   (b) 45   (c) 60   (d) 90

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x+y) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$$

$$\tan(x+y) = 1$$

$$x + y = \tan^{-1}(1) = 45°$$

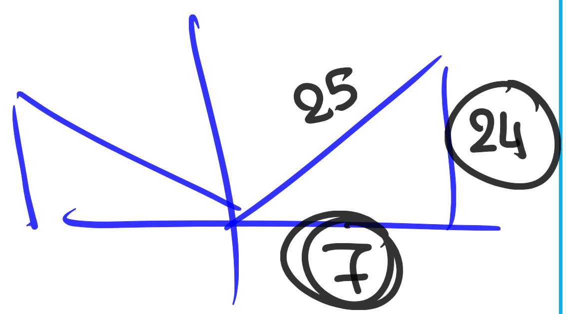
If $\sin x \cos x = \frac{12}{25}$, then $\cos 2x =$ ....

(a) $\frac{24}{25}$ (b) $\pm\frac{24}{25}$ (c) $\frac{7}{25}$ (d) $\pm\frac{7}{25}$

$\boxed{2 \sin x \cos x} = \frac{12}{25} \times 2$

$\sin 2x = \frac{24}{25}$

$\cos 2x = \pm \frac{7}{25}$



$$\frac{1-\tan^2 x}{1+\tan^2 x} = \dots\dots\dots\dots$$

(a) $\cos 2x$ (b) $\sin 2x$ (c) $\cos x$ (d) $\sin x$

$$\frac{1-\tan^2 x}{\sec^2 x} = \frac{1}{\sec^2 x} - \frac{\tan^2 x}{\sec^2 x}$$

$$= \frac{1}{\frac{1}{\cos^2 x}} - \frac{\frac{\sin^2 x}{\cancel{\cos^2 x}}}{\frac{1}{\cancel{\cos^2 x}}}$$

$$= \cos^2 x - \sin^2 x$$

$$= \cos 2x$$

If $\sin^2 2x + 4\sin x\cos x + 1 = 0$, $\frac{\pi}{2} < x < \pi$, then $x =$ $135 \times \frac{\pi}{180}$

(a) $\frac{7\pi}{12}$ (b) $\frac{2\pi}{3}$ 90 180 (c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{6}$

$\pi < 2x < 2\pi$

$\sin^2 2x + 2\left[2\sin x\cos x\right] + 1 = 0$

$\sin^2 2x + 2\sin 2x + 1 = 0$

$\sin 2x = -1$

3rd $180 + 90 = 270$ 135

4th $360 - 90 = 270$ 135

$3\frac{\pi}{2}$

If P is half the perimeter of the triangle ABC , P + a = 35 cm. , P + b = 34 cm. , c = 15 cm. , then the area of triangle ABC = ... cm^2.

21 21

(a) 64 (b) 72 (c) 84 (d) 96

$P + a + P + b + c = 35 + 34 + 15$

$Per + Per = 84 \Rightarrow Per. = 42$

$P = 21$

$a = 14$
$b = 13$
$c = 15$

$$A = \sqrt{P(P-a)(P-b)(P-c)}$$

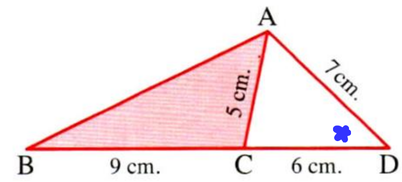
$$= \sqrt{21(21-14)(21-13)(21-15)} = 84$$

In the opposite figure :

The area of Δ ABC = ………….. cm^2.

(a) $6\sqrt{6}$ (b) $9\sqrt{6}$

(c) $9\sqrt{3}$ (d) $2\sqrt{6}$



$$\cos D = \frac{a^2 + c^2 - d^2}{2ac} = \frac{36 + 49 - 25}{2(6)(7)} = \frac{5}{7}$$

Area of Shaded $= \frac{1}{2} \times 7 \times 15 \times \sin D$

$- \frac{1}{2} \times 7 \times 6 \times \sin D$

$(\sin x \cos x)\ (1 + \tan^2 x)\ (1 - \sin^2 x) = \cdots\cdots\cdots\cdots$

(a) 1 (b) $\sin 2x$ (c) $\frac{1}{2} \sin 2x$ (d) $2 \sin 2x$

$\left(\frac{1}{2}\sin 2x\right) \cancel{\sec^2 x} \times \cancel{\cos^2 x}$

$\frac{1}{2} \sin 2x$

$A+B=180$

If $\angle$ A supplements $\angle$ B , then cos A cos B – sin A sin B = ...

(a) zero (b) 1 (c) – 1 (d) 2

$\cos(A+B)$

$\cos 180 = -1$

The slope of tangent to the curve y = sin 2 x at $x = \frac{\pi}{2}$ equals ……………

(a) 1 (b) zero (c) – 1 (d) – 2

$y = \sin 2x$

$\frac{dy}{dx} = 2 \cos 2x$

$\left.\frac{dy}{dx}\right]_{x=90} = 2 \cos(180) = -2$

If $y = (\sin x + \cos x)^2$, then $\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} =$ ....

(a) 2 (b) – 2 (c) 0 (d) – 1

$y = 1 + \sin 2x$

$\frac{dy}{dx} = 2\cos 2x$

$\left.\frac{dy}{dx}\right]_{x=\frac{\pi}{4}} = 2\cos 90 = \text{Zero}$

If $f(x) = \tan(5x - \pi)$, then $\grave{f}\left(\frac{\pi}{4}\right) = \dots\dots\dots\dots$

(a) 5 (b) $5\sqrt{2}$ (c) 10 (d) $10\sqrt{3}$

$\grave{f}(x) = 5 \sec^2(5x - \pi)$

$= 5 \sec^2 45$

$= 5 \times 2 = 10$

$$\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{4} + h\right) - \sin\left(\frac{\pi}{4}\right)}{h} = \ldots\ldots\ldots\ldots$$

(a) $\frac{\sin h}{h}$ (b) $\sin h$ (c) $\cos h$ (d) $\cos \frac{\pi}{4}$

Rate of change

Sin x

$$\boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$y = \sin x \Rightarrow y' = \cos x$

$= \cos \frac{\pi}{4}$

If $y = \sin^2 x + \cos^2 x$, then $\frac{dy}{dx} = \cdots\cdots\cdots\cdots$

(a) $2 \sin x \cos x$ (b) $\cos 2x$ (c) zero (d) 1

$y = 1 \Longrightarrow \frac{dy}{dx} = \text{zero}$

$\int \frac{d}{d\,x}(5 + \sin x)\, d\,x = $ .... $5x - \cos x + C$

(a) $5\,x - \cos x$ (b) $5\,x + \sin x + c$ (c) $\sin x + c$ (d) $5\,x - \cos x + c$

$\int \sin^2 x \sin 2x \, d x =$ .... $+ c$

(a) $\sin^2 x$ (b) $\sin 2x$ (c) $\frac{1}{2}\sin^3 x$ (d) $\frac{1}{2}\sin^4 x$

$\int \sin^2 x \cdot (2 \sin x \underbrace{\cos x}_{[f(x)]^3}) \, dx$

$\cancel{f'(x)}$

$2 \int \cancel{\cos x} \, [\sin x]^3 \, dx$

$2 \cdot \frac{\sin^4 x}{4} = \frac{1}{2} \sin^4 x + c$

$\int \tan^7 x \cot^7 x \, dx =$ …………… $+ c$

(a) $-x + \tan x$ (b) $x + \tan x$ (c) $-x + \cot x$ (d) x

$\int (\sin^2 x + \cos^2 x)\, dx =$ ……………

(a) $\frac{1}{3}\sin^3 x + \frac{1}{3}\cos^3 x + c$

(b) $-\cos^2 x + \sin^2 x + c$

(c) $\sec^2 x + c$

(d) $x + c$

Find the equation of the tangent to the curve of the function $f : f(X) = 2 \tan X - \cos^2 X$ at the point $(0 , -1)$.

Find the tangent equation to the curve of : $y = 2x \sin x \cos x$ when $x = \pi$

Best wishes

Mr. Michael Gamil